

Review Quick Hit (2)

1. Find the solution  $y(x)$  with  $y(0) = 4$  of the differential equation

$$\frac{dy}{dx} = x^2 y$$

$$\rightarrow \frac{dy}{y} = x^2 dx \rightarrow \int \frac{dy}{y} = \int x^2 dx \rightarrow \ln(|y|) = \frac{x^3}{3} + C$$

$$\rightarrow |y| = e^{\frac{x^3}{3} + C} = e^{\frac{x^3}{3}} e^C = k e^{\frac{x^3}{3}}$$

$$\rightarrow 4 = k e^0 = k$$

$$\rightarrow |y| = 4 e^{\frac{x^3}{3}}$$

2. Find the average value of the function  $g(x) = \frac{x^2}{x^3-1}$  on the interval  $[2, 4]$

$$\frac{1}{4-2} \int_2^4 \frac{x^2}{x^3-1} dx = \frac{1}{2} \int_{x=2}^{x=4} \frac{1}{u} \frac{du}{3} = \frac{1}{6} \ln(|u|) \Big|_{x=2}^{x=4}$$

$$(u = x^3 - 1; \frac{du}{3} = x^2 dx)$$

$$= \frac{1}{6} \ln(|x^3 - 1|) \Big|_2^4$$

$$= \frac{1}{6} \ln(63) - \frac{1}{6} \ln(7)$$

3. Compute  $\int_0^\infty x e^{-x^2} dx$

$$= \lim_{b \rightarrow \infty} \int_0^b x e^{-x^2} dx = \lim_{b \rightarrow \infty} \int_{x=0}^{x=b} e^u \frac{du}{-2} = \lim_{b \rightarrow \infty} -\frac{1}{2} e^u \Big|_{x=0}^{x=b}$$

$$(u = -x^2; \frac{du}{-2} = x dx)$$

$$= \lim_{b \rightarrow \infty} -\frac{1}{2} e^{-x^2} \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} -\frac{1}{2} e^{-b^2} + \frac{1}{2} e^0$$

$$= \boxed{\frac{1}{2}}$$