

# Midterm 1 Review

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## General Information

The exam will take place on Thursday, April 26 at 3:00 pm in Deady 205 (at the usual time in the usual classroom). You will be permitted to use a *scientific calculator only*, though a calculator will not be necessary to solve many of the problems on the exam. You must bring your own calculator if you wish to use one. You will not be permitted to use notes of any kind during the exam. In addition to using this review guide, you should study the example problems we've done in class, homework problems, and previous quiz problems. You should be prepared to answer questions about...

## Topics

- Parent functions
  - Domain
  - Image
  - Long-term behavior
- Even and odd functions
- Vertical transformations
- Horizontal transformations
- Combinations of vertical and horizontal transformations
- Point-slope form of a line
- Transformations of graphs of equations
  - Transforming the circle
- Periodic functions
  - Period
  - Amplitude
  - Midline
  - Computations
  - Graphs

## Midterm 1 Review

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### Practice Problems

True or False Questions:

1. True or False: If  $f(-2) = -f(2)$ , then  $f$  must be an odd function.

*False*

2. True or False: The graph of  $g(x) = e^{x+1}$  is a shift to the left of the graph of  $e^x$  by one unit.

*True*

3. True or False: If the domain of the function  $f(x)$  is  $[2, \infty)$ , then the domain of  $g(x) = f(2x)$  is  $[4, \infty)$ .

*False (the domain is actually  $[1, \infty)$ )*

4. True or False: If the domain of the function  $f(x)$  is  $[2, \infty)$ , then the domain of  $g(x) = 2f(x)$  is  $[2, \infty)$ .

*True*

5. True or False: Let  $g(x) = f(-x + 1)$ . Then  $g$  is the transformation that first reflects the graph of  $f$  across the vertical axis and then shifts the resulting graph left by 1.

*False (  $g(x) = f(-x+1) = f(-(x-1)) \rightarrow$  reflect then shift right )*

## Midterm 1 Review

### Short Answer Questions:

1. Are the following functions even, odd, neither, or both?

(a)  $f(x) = \frac{x^2 + x^3 + 1}{x}$

$$f(-x) = \frac{(-x)^2 + (-x)^3 + 1}{(-x)} = \frac{x^2 - x^3 + 1}{-x} \neq f(x) \\ \neq -f(x)$$

so  $f(x)$  is neither even, nor odd

(b)  $g(x) = xe^{x^2+2} - x^3 \log_3(|x|)$

$$g(-x) = (-x)e^{(-x)^2+2} - (-x)^3 \log_3(|-x|)$$

$$= -xe^{x^2+2} + x^3 \log_3(|x|)$$

$$= -\left(xe^{x^2+2} - x^3 \log_3(|x|)\right)$$

$$= -g(x) \rightarrow \text{so } g(x) \text{ is } \underline{\text{odd}}$$

(c)  $h(x) = 0$

$$h(-x) = 0$$

$$= h(x)$$

$$= -h(x)$$

$\rightarrow h(x)$  is both even and odd

(d)  $k(x) = e(x)d(x)$  where  $e$  is even and  $d$  is odd.

$$k(-x) = e(-x)d(-x)$$

$$= (e(x))(-d(x))$$

$$= -e(x)d(x)$$

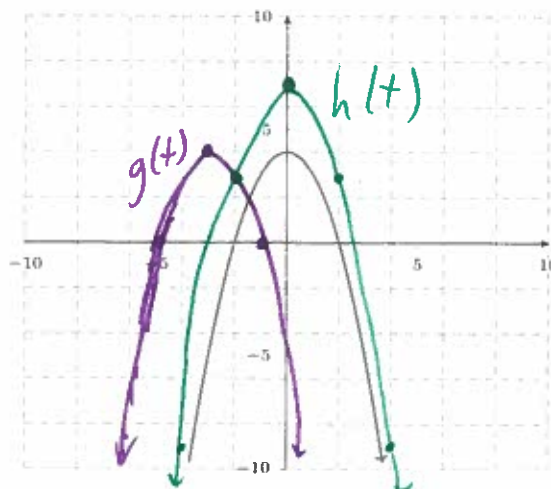
$$= -k(x) \rightarrow k(x) \text{ is odd}$$

## Midterm 1 Review

2. Let  $g$  be the function obtained from  $f$  by a vertical shift up 3 units, followed by a horizontal shift left 1 unit. Write an equation for  $g$  in terms of  $f$ .

$$g(x) = f(x+1) + 3$$

3. The graph of  $f(t)$  is shown below. Sketch and label the graphs of  $g(t) = f(t+3)$  and  $h(t) = f(t) + 3$ .

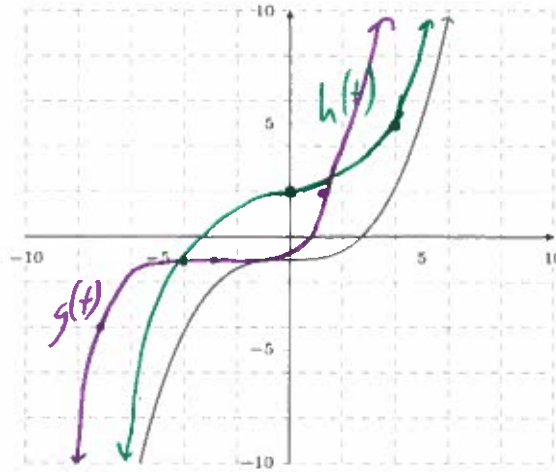


4. Let  $g$  be the function obtained from  $f$  by a reflection in the  $y$ -axis, followed by a vertical shift down 2 units, followed by a reflection in the  $x$ -axis. Write an equation for  $g$  in terms of  $f$ .

① <sup>horiz</sup> reflection  $\rightarrow f(-x)$   
② shift down 2  $\rightarrow f(-x) - 2$   
③ vertical reflection  $\rightarrow - (f(-x) - 2)$

## Midterm 1 Review

5. The graph of  $f(t) = 0.05t^3 - 1$  is shown below. Sketch and label the graph of  $g(t) = f(t+3)$  and  $h(t) = f(t) + 3$ .



6. Describe a sequence of graph transformations taking the function  $f(t) = t^2$  to the function  $g(t) = (t-2)^2 + 3$ .

① shift right 2 units  
② shift up 3 units

7. Describe a sequence of graph transformations taking the function  $f(t) = e^t$  to the function  $g(t) = e^{t+5} - 1$ . What is the range of  $g(t)$ ?

① shift left 5 units | The range of  $f(t)$  is  $(0, \infty)$   
② shift down 1 unit | The range of  $g(t)$  is the range of  $f$  shifted down 1, so  $(-1, \infty)$

8. Let  $g(t)$  be the function obtained from  $f(t) = e^{0.05t} - 2$  by a reflection in the vertical axis, followed by a vertical shift up 2 units. Write an equation for  $g(t)$ .

$$\begin{aligned} g(t) &= e^{-0.05t} - 2 + 2 \\ &= e^{-0.05t} \end{aligned}$$

## Midterm 1 Review

9. For each of the following functions, identify a possible parent function, and list all transformations (in the correct order) to get from the parent function to the desired function.

$$p(x) = x^2 \leftarrow$$

$$(a) f(x) = -(4x - 2)^2 + 3 = -\left(4\left(x - \frac{1}{2}\right)\right)^2 + 3 \rightarrow \begin{matrix} A = -1 & h = 1/2 \\ B = 4 & k = 3 \end{matrix}$$

- ① vertical reflection
- ② horizontal stretch by a factor of  $1/4$
- ③ shift right  $1/2$  unit
- ④ shift up 3 units

$$(b) h(x) = x^3 + 3x^2 + 3x + 1 = (x + 1)^3 \rightarrow \begin{matrix} A = 1 & h = -1 \\ B = 1 & k = 0 \end{matrix}$$

$$p(x) = x^3$$

- ① shift left 1 unit

$$(c) k(x) = 2^{x-1} \xrightarrow{\text{rewrite}} = (e^{\ln(2)})^{x-1} = e^{\ln(2)(x-1)}$$

$$p(x) = e^x \rightarrow \begin{matrix} A = 1 & h = 1 \\ B = \ln(2) & k = 0 \end{matrix}$$


- ① horizontal stretch by a factor of  $1/\ln(2)$
- ② shift right 1 unit

## Midterm 1 Review

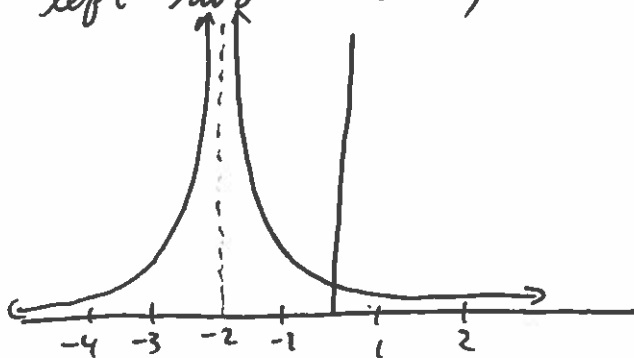
### Free Response Questions:

1. Sketch a graph of the following functions


(a)  $g(x) = \frac{1}{(x+2)^2}$

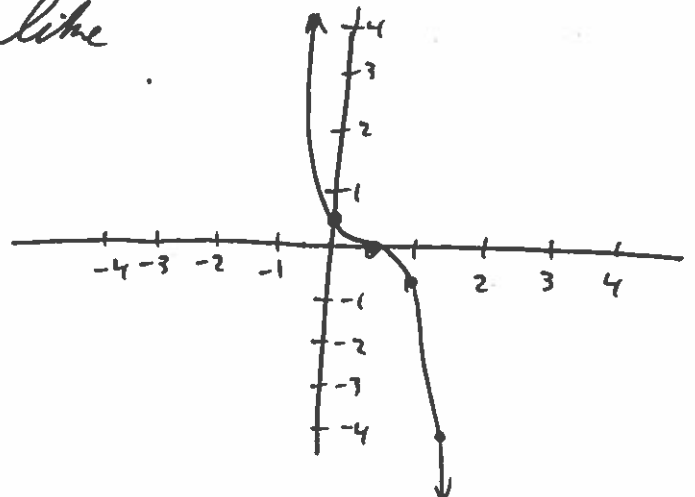
parent function is  $\frac{1}{x^2}$ ,  
which has graph 

So the graph of  $g(x)$  is a shift of  $\frac{1}{x^2}$   
left two units, so  $g(x)$  looks like



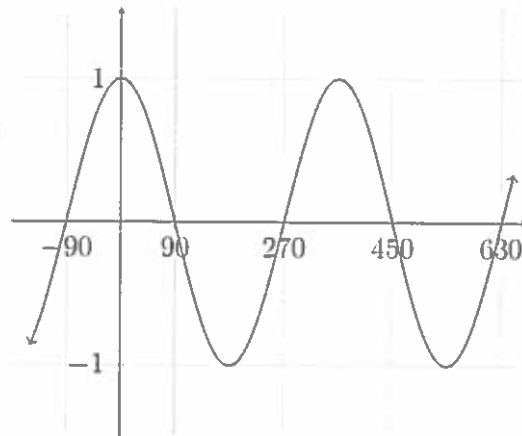
(b)  $h(x) = \frac{1}{2}(1-2x)^3 = \frac{1}{2}(-2(x-\frac{1}{2}))^3$

parent function is  $p(x) = x^3$ , which has graph   
we have a vertical stretch by a factor of  $\frac{1}{2}$ ,  
a horizontal stretch by a factor of  $-\frac{1}{2}$ ,  
then a shift right by  $\frac{1}{2}$  units, so our  
new graph looks like



## Midterm 1 Review

2. Let  $f$  be given by the graph below.

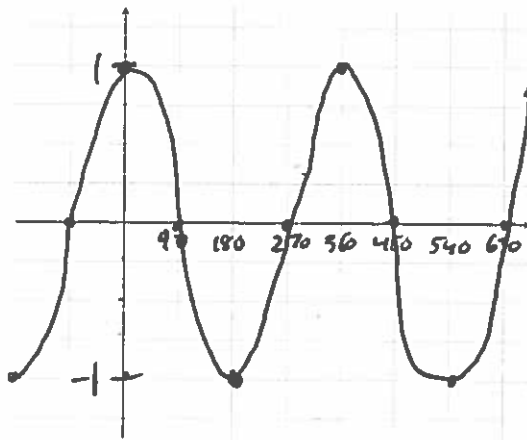


Sketch the following transformations.

(a)  $-f(x - 2)$

typo: this should be  $-f(x - 180)$ , which has the following graph

- vertical reflection
- shift right 180 units

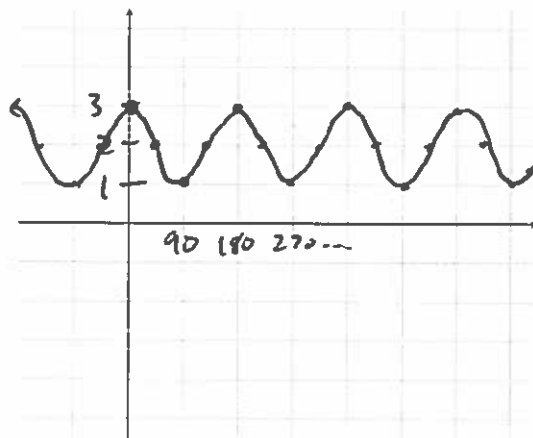




## Midterm 1 Review

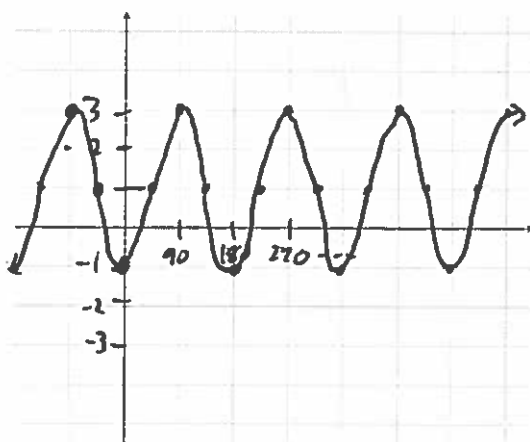
(b)  $f(2x) + 2$

- having stretch by a factor of  $\frac{1}{2}$
- shift up 2



(c)  $2f\left(\frac{x+90}{2}\right) + 1$

- vertical stretch by factor of 2
- horizontal stretch by a factor of 2
- shift left 90
- shift up 1

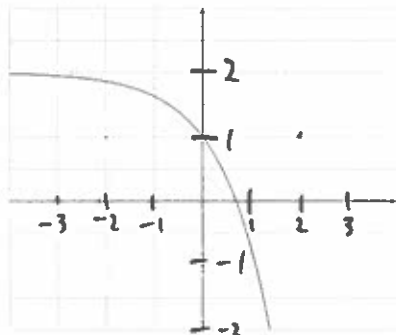


## Midterm 1 Review

3. Find equations for each of the following graphs given the parent function  $p(x)$

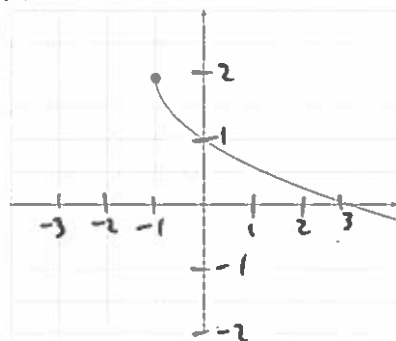
(a)  $p(x) = e^x$

typo: I should  
have labeled  
the axes here...



$$f(x) = -e^x + 2$$

(b)  $p(x) = \sqrt{x}$



$$f(x) = -\sqrt{x+1} + 2$$

## Midterm 1 Review

4. Suppose that  $f$  is periodic with period 3, where  $f(x) = \ln(x + \frac{1}{2})$  on the interval  $[0, 3)$ .

(a) Find all roots of  $f$ .  $f(x) = 0 \rightarrow \ln(x + \frac{1}{2}) = 0 \rightarrow x + \frac{1}{2} = e^0 = 1$   
 $\rightarrow x = \frac{1}{2}$

on  $[0, 3)$

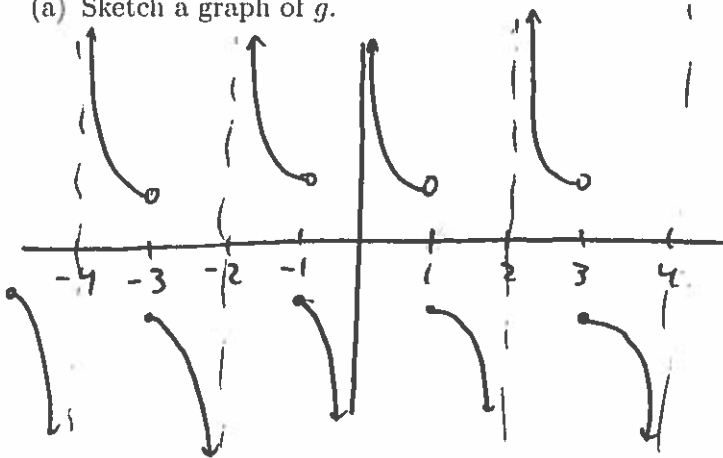
So all solutions have the form  $\frac{1}{2} + 3n$  for integers  $n$

- (b) Compute  $f(100)$ .

$$f(100) = f(100 - 3(33)) = f(1) = \ln(1.5)$$

5. Suppose that  $g$  is periodic with period 2, where  $g(x) = 1/x$  on the interval  $[-1, 1)$ .

- (a) Sketch a graph of  $g$ .



- (b) What is the midline and amplitude of  $g$ ?

There is no midline and there is no amplitude because  $g(x)$  has no maximum and no minimum  $y$ -value

