Homework 4

1. *Show that* $8n + 3$ *and* $5n + 2$ *are relatively prime for all integers* n.

   Your answer here...

2. *Using Euclid's proof that there are infinitely many primes (this is the first proof we gave in class), show that the* n*th prime,* p_n*, does not exceed* $2^{2^{n-1}}$ *for* $n \geq 1$*. Conclude that when* n *is a positive integer, there are at least* $n+1$ *primes less than* 2^{2^n}*. Conclude that for integers* x *of the form* 2^{2^n}*,* $\pi(x) \geq \log_2 \log_2 x$*.*

   Hint: if you're having a hard time with this one, it may be helpful to do problem 4a first.

   Your answer here...

3. *Show that if* $n \in \mathbb{Z}_{>1}$ *and* $i, j \in \mathbb{N}$ *with* $1 \leq i < j \leq n$, *then*

$$(n! \cdot i + 1, n! \cdot j + 1) = 1$$

Hint: You may use the fact that if p *is prime and* $p \mid ab$, *then* $p \mid a$ *or* $p \mid b$.

Your answer here...

4. *Show that if* $a^k - 1$ *is prime (with* $a \geq 1$ *and* $k \geq 2$*), then...*

   (a) $\ldots a = 2$

   Your answer here...

   (b) $\ldots k$ *is prime*

   Hint: Use a computer or website (e.g. type "factor 15" into Wolfram Alpha) to find some prime factors of $2^k - 1$ *when* k *isn't prime for several values of* k *(try a few even composite values of* k *and also* $k = 15$*). If you have a hard time proving this, you may make a conjecture about which sorts of numbers divide* $2^k - 1$ *when* k *is not prime and provide some supporting evidence for partial credit.*

   Your answer here...