

Objective: To practice using Wilson's Theorem and Fermat's Little Theorem and to preview the concepts of quadratic residues.

1. What is the remainder when $18!$ is divided by 437?

2. What is the remainder when 5^{100} is divided by 7?

3. Find the least positive residue of $3^{999,999,999}$ modulo 7.

4. Use Fermat's Little Theorem to solve $7x \equiv 12 \pmod{17}$

5. **Def:** If $m > 0$, we say that a is a quadratic residue of m if $(a, m) = 1$ and the congruence $x^2 \equiv a \pmod{m}$ has a solution.

(a) Find all quadratic residues of 7. Pick your favorite quadratic residue a which is not a perfect square (in \mathbb{Z}) and find all solutions to $x^2 \equiv a \pmod{7}$

(b) Find all quadratic residues of 24. Pick your favorite quadratic residue a which is not a perfect square (in \mathbb{Z}) and find all solutions to $x^2 \equiv a \pmod{24}$