

Objective: The goal of this worksheet is to help you become more comfortable working with prime numbers and greatest common divisors.

1. Show that if  $k \in \mathbb{Z}_{>0}$ , then  $3k + 2$  and  $5k + 3$  are relatively prime.

2. Show that if  $n \in \mathbb{Z}_{>0}$ , then  $(n+1, n^2 - n + 1) = 1$  or  $3$

3. We say that integers  $a_1, \dots, a_n$  are mutually relatively prime if  $(a_1, \dots, a_n) = 1$ . The integers are pairwise relatively prime if  $(a_i, a_j) = 1$  when  $i \neq j$ .

(a) Can you find four integers which are mutually relatively prime so that any three of them are not mutually relatively prime?

(b) Can you find four integers which are pairwise relatively prime so that any three of them are not mutually relatively prime?

4. Write  $(630, 156)$  as a linear combination of 630 and 156 in two different ways.

5. Suppose that  $n$  is a positive integer and let  $p$  be the smallest prime factor of  $n$ . Show that if  $p > n^{1/3}$ , then  $\frac{n}{p}$  is either prime or equal to 1.

6. Suppose that  $p$  is prime. Show that if  $p \mid ab$ , then  $p \mid a$  or  $p \mid b$ .