Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- Remember to let me know if you can't do your peer review this week (for any reason)!
- A helpful fact for this assignment: $x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \cdots + x + 1)$

Here's the schedule of when things are due for this assignment.

Component Due	**Date**
Draft of Solutions	Thursday, January 27
Peer Review	Tuesday, February 1
Final Copy	Friday, February 4

And finally, here are the problem statements.

1. *Show that $8n+3$ and $5n+2$ are relatively prime for all integers n.*

2. *Using Euclid's proof that there are infinitely many primes (this is the first proof we gave in class), show that the nth prime, p_n, does not exceed $2^{2^{n-1}}$ for $n \geqslant 1$. Conclude that when n is a positive integer, there are at least $n+1$ primes less than 2^{2^n}. Conclude that for integers x of the form 2^{2^n}, $\pi(x) \geqslant \log_2 \log_2 x$.*

   Hint: if you're having a hard time with this one, it may be helpful to do problem 4a first.

3. *Show that if $n \in \mathbb{Z}_{>1}$ and $i, j \in \mathbb{N}$ with $1 \leqslant i < j \leqslant n$, then*

$$(n! \cdot i + 1, n! \cdot j + 1) = 1$$

   Hint: You may use the fact that if p is prime and $p \mid ab$, then $p \mid a$ or $p \mid b$.

4. *Show that if $a^k - 1$ is prime (with $a \geqslant 1$ and $k \geqslant 2$), then...*

   (a) $\ldots a = 2$

   (b) $\ldots k$ *is prime*

   Hint: Use a computer or website (e.g. type "factor 15" into Wolfram Alpha) to find some prime factors of $2^k - 1$ when k isn't prime for several values of k (try a few even composite values of k and also $k = 15$). If you have a hard time proving this, you may make a conjecture about which sorts of numbers divide $2^k - 1$ when k is not prime and provide some supporting evidence for partial credit.