1. True or false?

   (a) **True** or **False**? $20 \equiv 38 \mod 4$

   (b) **True** or **False**? $-9 \equiv -5 \mod 4$

   (c) **True** or **False**? $15 \equiv 2 \mod 0$

   (d) **True** or **False**? $81 \equiv -92 \mod 1$

2. Prove the following statements about congruences

   (a) For all $a \in \mathbb{Z}$, $a \equiv a \mod m$

   (b) For all $a, b \in \mathbb{Z}$, $a \equiv b \mod m$ if and only if $b \equiv a \mod m$

   (c) For all $a, b, c \in \mathbb{Z}$, if $a \equiv b \mod m$ and $b \equiv c \mod m$, then $a \equiv c \mod m$

   (d) If $a \equiv b \mod m$, then $a + c \equiv b + c \mod m$

   (e) If $a \equiv b \mod m$, then $ac \equiv bc \mod m$

3. Is it true that if $ac \equiv bc \mod m$ then $a \equiv b \mod m$?

4. Find all solutions to the diophantine equation $102x + 1001y = 1$. If there are none, modify the equation appropriately so that there is at least one solution and classify all solutions to that equation.

5. Let $a$ and $b$ be relatively prime positive integers and let $n$ be a positive integer. A solution $(x, y)$ of the linear diophantine equation $ax + by = n$ is <u>nonnegative</u> if both $x$ and $y$ are nonnegative. Show that whenever $n \geqslant (a-1)(b-1)$, there is a nonnegative solution of $ax + by = n$