Week 5 Group Work

1. Which positive integers have exactly three positive divisors? Which have exactly four positive divisors?

2. Let n be a positive integer. Show that the power of the prime p occurring in the prime-power factorization of $n!$ is
$$\lfloor \frac{n}{p} \rfloor + \lfloor \frac{n}{p^2} \rfloor + \lfloor \frac{n}{p^3} \rfloor + \cdots$$
Recall that $\lfloor x \rfloor$ is the largest integer less than or equal to x.

3. How many zeros are there at the end of 1000!? (the result from the previous problem is helpful here)

4. Show that $\sqrt{2}$ is irrational using the fundamental theorem of arithmetic.

5. Show that $\log_2 3$ is irrational