

Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- There won't usually be an extra credit problem. However, there is this time since this assignment is helping me assess what kind of background we have.
- Remember to let me know if you can't do your peer review this week (for any reason)!
- You are not required to use Overleaf. If you have L<sup>A</sup>T<sub>E</sub>X installed on a personal computer, I recommend using that.

Here's the schedule of when things are due for this assignment.

Component Due	Date
Draft of Solutions	Thursday, January 13
Peer Review	Tuesday, January 18
Final Copy	Friday, January 21

And finally, here are the problem statements.

1. Show that if  $a$  and  $b$  are positive integers, then there is a smallest positive integer of the form  $a - bk$  for  $k \in \mathbb{Z}$ .
2. Prove that  $7^n$  has the last two digits (in base 10)

$$\left\{ \begin{array}{l} 07 \text{ if } n \text{ is of the form } 4k + 1 \\ 49 \text{ if } n \text{ is of the form } 4k + 2 \\ 43 \text{ if } n \text{ is of the form } 4k + 3 \\ 01 \text{ if } n \text{ is of the form } 4k \end{array} \right.$$

3. What is wrong with the following proof?

**Proposition:** All horses are the same color.

*Proof.* By (strong) induction on the number of horses.

Base cases: This is true if there are zero horses. It is also true if there is only one horse.

Inductive step: We assume that the statement holds for any group of  $k$  horses (or smaller) and show that it holds for a group of  $k + 1$  horses. Suppose we have a group of  $k + 1$  horses. Choose one, call it Winnie. The group, minus Winnie, has only  $k$  horses, so those horses are all the same color by assumption. Now choose another horse, call it Tigger. The group, minus Tigger (but including Winnie), has  $k$  horses again, and so they are all the same color by assumption. The overlap,  $k - 1$  horses, are also all of the same color by assumption. Therefore, any group of horses are the same color. Since there are a finite number of horses in the world, they must all be of the same color.  $\square$

4. Find three different formulas or rules for the terms of a sequence  $\{a_n\}$  if the first three terms of the sequence are 1, 2, 4.

5. (Extra Credit). Let  $H_n$  be the  $n$ th partial sum of the harmonic series. I.e.  $H_n = \sum_{j=1}^n \frac{1}{j}$

- (a) Prove that  $H_{2^n} \geq 1 + \frac{n}{2}$
- (b) Prove that  $H_{2^n} \leq 1 + n$
- (c) Why does this imply that  $H_n \approx \log_2(n)$ ?