1. Find a counterexample to the following claim: "Let p be an odd prime and $a, b, c \in \mathbb{Z}_{>0}$ with c not a multiple of p. Suppose further that $c^a \equiv c^b \mod p$. Then $a \equiv b \mod p-1$."

2. Let $n \in \mathbb{Z}_{>0}$ and suppose n factors into prime powers as

$$n = p_1^{2t_1+1} p_2^{2t_2+1} \cdots p_k^{2t_k+1} p_{k+1}^{2t_{k+1}} \cdots p_m^{2t_m}$$

Show that if q is an odd prime not dividing n then

$$\left(\frac{n}{q}\right) = \left(\frac{p_1}{q}\right)\left(\frac{p_2}{q}\right) \cdots \left(\frac{p_k}{q}\right)$$

3. Show that if b is a positive integer then $\displaystyle\sum_{j=1}^{p-1} \left(\frac{jb}{p}\right) = 0$

4. Compute $\left(\frac{111}{991}\right)$. Make up your own Legendre symbol to compute (and don't forget that the "denominator" is required to be prime).

5. (a) Develop a congruence relation that describes exactly when 5 is a quadratic residue modulo an odd prime p. I.e. your answer should look something like "5 is a quadratic residue mod p if and only if $p \equiv a_1 \mod m$ or $p \equiv a_2 \mod m$ or ... or $p \equiv a_k \mod m$" where you replace $a_1, \ldots, a_k$ and m by appropriate numbers.

   (b) Develop a congruence relation that describes exactly when 7 is a quadratic residue modulo an odd prime p.