

1. *A grocer orders apples and oranges at a total cost of \$8.39. If apples cost 25 cents each and oranges cost 18 cents each, how many of each type of fruit did the grocer order? Is there more than one possibility? Explain.*

Your answer here...

2. Define the set

$$\mathbb{Z}/m\mathbb{Z} := \{0, 1, 2, \dots, m-1\}$$

(pronounced “zee mod m zee” in America and “zed mod m zed” in some other parts of the English-speaking world). For any  $a, b \in \mathbb{Z}/m\mathbb{Z}$ , define  $a + b$  to be the unique element in  $\mathbb{Z}/m\mathbb{Z}$  which is congruent to  $a + b$  (usual addition in  $\mathbb{Z}$ ) modulo  $m$ . Define  $a \cdot b$  to be the unique element in  $\mathbb{Z}/m\mathbb{Z}$  which is congruent to  $ab$  (usual multiplication in  $\mathbb{Z}$ ) modulo  $m$ .

(a) Fill out the following table of addition for  $\mathbb{Z}/5\mathbb{Z}$ . In the row corresponding to  $a$  and column corresponding to  $b$ , put  $a + b$ . You can just fill out the table, you don't have to show work here.

+	0	1	2	3	4
0					
1					
2					
3					
4					

(b) Fill out the following table of multiplication for  $\mathbb{Z}/5\mathbb{Z}$ . In the row corresponding to  $a$  and column corresponding to  $b$ , put  $a \cdot b$ . You can just fill out the table, you don't have to show work here.

·	0	1	2	3	4
0					
1					
2					
3					
4					

3. *Show that if  $a \equiv b \pmod{m}$  and  $d \mid m$ , then  $a \equiv b \pmod{d}$ . Show that it is not the case that if  $a \equiv b \pmod{d}$ , then  $a \equiv b \pmod{kd}$ .*

Your answer here...

4. *Suppose that  $a, b, c, d, m \in \mathbb{Z}$  with  $a \equiv b \pmod{m}$  and  $c \equiv d \pmod{m}$ . Show that  $ac \equiv bd \pmod{m}$ .*

Your answer here...

5. *Suppose that  $a, b, k, m \in \mathbb{Z}$  with  $k \geq 0$  and  $m \geq 1$ . Show that if  $a \equiv b \pmod{m}$ , then  $a^k \equiv b^k \pmod{m}$ . You may use the result of the previous problem even if you were unable to prove it.*

Your answer here...

6. Show that, for  $n \geq 0$ ,

$$7^n \equiv \begin{cases} 1 \pmod{100} & \text{if } n \equiv 0 \pmod{4} \\ 7 \pmod{100} & \text{if } n \equiv 1 \pmod{4} \\ 49 \pmod{100} & \text{if } n \equiv 2 \pmod{4} \\ 43 \pmod{100} & \text{if } n \equiv 3 \pmod{4} \end{cases}$$

Your answer here...