Homework 2

1. *Show that if a and b are positive integers, then there is a smallest positive integer of the form $a - bk$ for $k \in \mathbb{Z}$.*

   Your answer here...

2. *Prove that 7^n has the last two digits (in base 10)*

$$\begin{cases} 07 & \text{if } n \text{ is of the form } 4k+1 \\ 49 & \text{if } n \text{ is of the form } 4k+2 \\ 43 & \text{if } n \text{ is of the form } 4k+3 \\ 01 & \text{if } n \text{ is of the form } 4k \end{cases}$$

Your answer here...

3. *What is wrong with the following proof?*

Proposition: All horses are the same color.

Proof. By (strong) induction on the number of horses.

Base cases: This is true if there are zero horses. It is also true if there is only one horse.

Inductive step: We assume that the statement holds for any group of k horses (or smaller) and show that it holds for a group of $k+1$ horses. Suppose we have a group of $k+1$ horses. Choose one, call it Winnie. The group, minus Winnie, has only k horses, so those horses are all the same color by assumption. Now choose another horse, call it Tigger. The group, minus Tigger (but including Winnie), has k horses again, and so they are all the same color by assumption. The overlap, $k-1$ horses, are also all of the same color by assumption. Therefore, any group of horses are the same color. Since there are a finite number of horses in the world, they must all be of the same color. $\square$

Your answer here...

4. *Find three diferent formulas or rules for the terms of a sequence* $\{a_n\}$ *if the first three terms of the sequence are 1, 2, 4.*

Your answer here...

5. (Extra Credit). *Let H_n be the nth partial sum of the harmonic series. I.e.* $H_n = \displaystyle\sum_{j=1}^{n} \frac{1}{j}$

   (a) *Prove that* $H_{2^n} \geq 1 + \frac{n}{2}$

   (b) *Prove that* $H_{2^n} \leq 1 + n$

   (c) *Why does this imply that* $H_n \approx \log_2(n)$*?*