

Objective: The goal of this worksheet is to gain familiarity with properties of divisibility and see how and when induction can be helpful in proving statements about divisibility.

1. Show that every integer falls into one of the following categories:

- (a) Even: $n = 2j$ for some $j \in \mathbb{Z}$
- (b) Threven: $n = 3k$ for some $k \in \mathbb{Z}$
- (c) Plus one: $n = 6r + 1$ for some $r \in \mathbb{Z}$
- (d) Plus five: $n = 6s + 5$ for some $s \in \mathbb{Z}$

Are these categories disjoint?

2. Show that for all $n \in \mathbb{Z}$, $6 \mid n(n+1)(2n+1)$

3. Show that the product of two integers of the form $4k + 1$ is again of this form. Show that the product of two integers of the form $4k + 3$ is of the form $4k + 1$

4. The n th Fibonacci number is defined as follows:

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

(a) Compute the first 6 Fibonacci numbers.

(b) Show that F_n is even if and only if $3 \mid n$

5. Compute $(29, 11)$, $(100, 7)$, and $(-356, 16)$

6. For $a, b \in \mathbb{Z}$, what are $(a, 0)$, $(a, 1)$, (a, a) , and (a, ab) ?

7. Suppose that $a, b \in \mathbb{Z}$ and $(a, b) = 1$. Show that $(a + b, a - b) = 1$ or 2 . When is this GCD 1? When is it 2?