

Objective: The goal of this worksheet is to help you become more comfortable working with prime numbers and greatest common divisors.

1. Show that if $k \in \mathbb{Z}_{>0}$, then $3k + 2$ and $5k + 3$ are relatively prime.

2. Show that if $n \in \mathbb{Z}_{>0}$, then $(n+1, n^2 - n + 1) = 1$ or 3

3. We say that integers a_1, \dots, a_n are mutually relatively prime if $(a_1, \dots, a_n) = 1$. The integers are pairwise relatively prime if $(a_i, a_j) = 1$ when $i \neq j$.

- (a) Can you find four integers which are mutually relatively prime so that any three of them are not mutually relatively prime?

- (b) Can you find four integers which are pairwise relatively prime so that any three of them are not mutually relatively prime?

4. Write $(630, 156)$ as a linear combination of 630 and 156 in two different ways.

5. Suppose that n is a positive integer and let p be the smallest prime factor of n . Show that if $p > n^{1/3}$, then $\frac{n}{p}$ is either prime or equal to 1.

6. Suppose that p is prime. Show that if $p \mid ab$, then $p \mid a$ or $p \mid b$.