Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- Remember to let me know if you can't do your peer review this week (for any reason)!

Here's the schedule of when things are due for this assignment.

Component Due	**Date**
Draft of Solutions	Thursday, April 14
Peer Review	Tuesday, April 19
Final Copy	Friday, April 22

And finally, here are the problem statements.

1. *Define Liouville's function* $\lambda(n)$ *so that* $\lambda(1) = 1$ *and for* $n \geqslant 2$, $\lambda(n) = (-1)^{e_1+\cdots+e_g}$ *when the prime factorization of* n *is* $p_1^{e_1} \cdots p_g^{e_g}$. *Is* λ *multiplicative? Is* λ *completely multiplicative?*

2. *An arithmetic function* f *is said to be* <u>*additive*</u> *if* $f(mn) = f(m) + f(n)$ *for all relatively prime positive integers* m *and* n. f *is said to be* <u>*completely additive*</u> *if* $f(mn) = f(m) + f(n)$ *for all positive integers* m *and* n. *For any prime integer* p, *define the function* $v_p(n)$ *by defining*

$$v_p(n) := \max\{k \in \mathbb{N} : p^k \mid n\}$$

   (a) *Is* v_p *additive? Is it completely additive?*

   (b) *Show that for any positive integers* a *and* b,

$$v_p(a+b) \geqslant \min(v_p(a), v_p(b))$$

3. *Find all positive integers* n *with* $\sigma(n) = 12$.

4. *A positive integer* $n > 1$ *is* <u>*highly composite*</u> *if* $\tau(m) < \tau(n)$ *whenever* $m < n$.

   (a) *Find the first five highly composite numbers*

   (b) *Show that if* n *is highly composite and* m *is a positive integer with* $\tau(m) > \tau(n)$, *then there exists a highly composite integer* k *so that* $n < k \leqslant m$. *Conclude that there are infinitely many highly composite integers.*