Week 8 Group Work

1. Find the smallest positive integer n so that $a^n \equiv 1 \mod 144$ for all $a \in (\mathbb{Z}\,/\,144\mathbb{Z})^\times$

2. Find all values of n so that $\lambda(n) = 6$.

3. Let L be the line in the xy plane passing through the point $(-1,0)$ and having slope t. Find the coordinates of the other intersection point between L and the unit circle given by $x^2+y^2=1$.

4. Find all primitive Pythagorean triples (x, y, z) so that $z \leqslant 20$

5. Show that if (x, y, z) is a primitive Pythagorean triple, then one of x, y, z is divisible by 5.

Homework Problem Candidates

1. Suppose that k and n are positive integers. Show that the set $\{1^k, 2^k, 3^k, \ldots, (n-1)^k\}$ forms a complete set of residues modulo n if and only if n is square-free and $(k, \lambda(n)) = 1$.

2. (a) Suppose $f(x_1, \ldots, x_n)$ is a polynomial with integer coefficients. Show that if there exist integers $(k_1, \ldots, k_n)$ so that $f(k_1, \ldots, k_n) = 0$, then there exists a solution to $f(x_1, \ldots, x_n) \equiv 0 \mod m$ for every positive integer m. What is the contrapositive of this statement?

   (b) Show that there are no solutions in integers to $x^2 + y^2 = 3z^2$

3. Classify all right triangles whose sides have integer lengths and whose area equals its perimeter.