

1. Which positive integers have an odd number of positive divisors?

2. What is the product of all positive divisors of a positive integer n ?

3. Define the Möbius function, $\mu : \mathbb{Z}_{>0} \rightarrow \mathbb{N}$ so that

$$\mu(n) = \begin{cases} 1 & n = 1 \\ (-1)^r & n = p_1 \cdots p_r \text{ where } p_1, \dots, p_r \text{ are distinct primes} \\ 0 & \text{otherwise} \end{cases}$$

Show that $\mu(n)$ is multiplicative.

4. Compute $p(6)$, $p^D(6)$, and $p_O(6)$ where O is the set of positive odd integers.

5. Show that for all $n \geq 1$,

$$p(n) = p(n-1) + p_2(n)$$