1. *Suppose that m and k are positive integers and that k is relatively prime to $\phi(m)$. Suppose also that m has a primitive root. Use Theorem 9.17 (or other methods) to show that the function*

$$f : \left(\mathbb{Z}\big/m\mathbb{Z}\right)^{\times} \to \left(\mathbb{Z}\big/m\mathbb{Z}\right)^{\times}$$
$$x \mapsto x^k$$

is injective.

Your answer here...

2. *Suppose that k and n are positive integers. In this problem, you will show that the set*

$$S = \{0, 1^k, 2^k, 3^k, \ldots, (n-1)^k\}$$

forms a complete set of residues modulo n if n is square-free and $(k, \lambda(n)) = 1$. The converse is true too, but I won't make you show that here.

(a) *Show that the only element of S which is congruent to 0 modulo n is 0.*

Your answer here...

(b) *Suppose $1 \leq x, y \leq n-1$ and p is a prime factor of n. Show that if $x^k \equiv y^k \mod n$, then $x \equiv y \mod p$.*

Your answer here...

(c) *Conclude that S forms a complete set of residues modulo n.*

Your answer here...

3. (a) *Suppose* $f(x_1, \ldots, x_n)$ *is a polynomial with integer coefficients. Show that if there exist integers* $(k_1, \ldots, k_n)$ *so that* $f(k_1, \ldots, k_n) = 0$*, then there exists a solution to* $f(x_1, \ldots, x_n) \equiv 0 \mod m$ *for every positive integer* m*. What is the contrapositive of this statement?*

   Your answer here...

   (b) *Show that there are no solutions in integers to* $x^2 + y^2 = 3z^2$

   Your answer here...

4. *Classify all right triangles whose sides have integer lengths and whose area equals its perimeter.*