

1. Which positive integers have an odd number of positive divisors?

2. What is the product of all positive divisors of a positive integer  $n$ ?

3. Define the Möbius function,  $\mu : \mathbb{Z}_{>0} \rightarrow \mathbb{N}$  so that

$$\mu(n) = \begin{cases} 1 & n = 1 \\ (-1)^r & n = p_1 \cdots p_r \text{ where } p_1, \dots, p_r \text{ are distinct primes} \\ 0 & \text{otherwise} \end{cases}$$

Show that  $\mu(n)$  is multiplicative.

4. Compute  $p(6)$ ,  $p^D(6)$ , and  $p_O(6)$  where  $O$  is the set of positive odd integers.

5. Show that for all  $n \geq 1$ ,

$$p(n) = p(n-1) + p_2(n)$$