1. Let $S = \{k > 0 : k \equiv 1 \mod 3\}$. Find an infinite product for the generating function of $p_S(n)$. Expand that product to find $p_S(n)$ for the first several n (up until $n = 9$ish)

2. What is the generating function for $p(n \mid \text{parts are distinct powers of 2})$?

3. Show that if n is a positive integer, then

$$p(n) \leqslant \frac{p(n-1) + p(n+1)}{2}$$

Hint: It may be helpful to use the fact from last week's worksheet that $p(n) = p(n-1) + p_2(n)$. This fact (probably) won't be immediately helpful though...