Homework 5

1. *Suppose that r is a primitive root mod p for an odd prime p. Show that*

$$r^{\frac{p-1}{2}} \equiv -1 \mod p$$

Your answer here...

2. *Suppose that p is an odd prime greater than 3 and S is a set of $\varphi(p-1)$ primitive roots modulo p. What is the least positive residue of the product of all elements of S modulo p?*

Your answer here...

3. *Suppose that p is prime and $p = 2q + 1$ where q is an odd prime.*

   (a) *How many primitive roots are there mod p?*

   Your answer here...

   (b) *Show that for any positive integer n with $1 < n < p-1$, the integer $-n^2$ is a primitive root modulo p.*

   Your answer here...

   (c) *Why do the answers to the previous two parts appear like they might contradict each other? Why do they not actually contradict each other?*

   Your answer here...

Homework 5

4. *Show that if the integer m has a primitive root then the only solutions to the congruence $x^2 \equiv 1 \mod m$ are $x \equiv \pm 1 \mod m$.*

   Your answer here...