Homework 3

1. *Define Liouville's function* $\lambda(n)$ *so that* $\lambda(1) = 1$ *and for* $n \geq 2$, $\lambda(n) = (-1)^{e_1+\cdots+e_g}$ *when the prime factorization of* n *is* $p_1^{e_1} \cdots p_g^{e_g}$. *Is* λ *multiplicative? Is* λ *completely multiplicative?*

   Your answer here...

2. *An arithmetic function f is said to be* <u>additive</u> *if $f(mn) = f(m)+f(n)$ for all relatively prime positive integers m and n. f is said to be* <u>completely additive</u> *if $f(mn) = f(m)+f(n)$ for all positive integers m and n. For any prime integer p, define the function $v_p(n)$ by defining*

$$v_p(n) := \max\{k \in \mathbb{N} : p^k \mid n\}$$

   (a) *Is v_p additive? Is it completely additive?*

   Your answer here...

   (b) *Show that for any positive integers a and b,*

$$v_p(a+b) \geq \min(v_p(a), v_p(b))$$

   Your answer here...

3. *Find all positive integers n with $\sigma(n) = 12$.*

Your answer here...

4. *A positive integer $n > 1$ is highly composite if $\tau(m) < \tau(n)$ whenever $m < n$.*

   (a) *Find the first five highly composite numbers*

   Your answer here...

   (b) *Show that if n is highly composite and m is a positive integer with $\tau(m) > \tau(n)$, then there exists a highly composite integer k so that $n < k \leq m$. Conclude that there are infinitely many highly composite integers.*

   Your answer here...