Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- There won't usually be an extra credit problem. However, there is this time since this assignment is helping me assess what kind of background we have.
- Remember to let me know if you can't do your peer review this week (for any reason)!
- You are not required to use Overleaf. If you have LaTeX installed on a personal computer, I recommend using that.

Here's the schedule of when things are due for this assignment.

Component Due	**Date**
Draft of Solutions	Thursday, January 13
Peer Review	Tuesday, January 18
Final Copy	Friday, January 21

And finally, here are the problem statements.

1. *Show that if a and b are positive integers, then there is a smallest positive integer of the form $a - bk$ for $k \in \mathbb{Z}$.*

2. *Prove that 7^n has the last two digits (in base 10)*

$$\begin{cases} 07 & \text{if } n \text{ is of the form } 4k+1 \\ 49 & \text{if } n \text{ is of the form } 4k+2 \\ 43 & \text{if } n \text{ is of the form } 4k+3 \\ 01 & \text{if } n \text{ is of the form } 4k \end{cases}$$

3. *What is wrong with the following proof?*

   Proposition: All horses are the same color.

   Proof. By (strong) induction on the number of horses.

   Base cases: This is true if there are zero horses. It is also true if there is only one horse.

   Inductive step: We assume that the statement holds for any group of k horses (or smaller) and show that it holds for a group of $k+1$ horses. Suppose we have a group of $k+1$ horses. Choose one, call it Winnie. The group, minus Winnie, has only k horses, so those horses are all the same color by assumption. Now choose another horse, call it Tigger. The group, minus Tigger (but including Winnie), has k horses again, and so they are all the same color by assumption. The overlap, $k-1$ horses, are also all of the same color by assumption. Therefore, any group of horses are the same color. Since there are a finite number of horses in the world, they must all be of the same color. □

4. *Find three diferent formulas or rules for the terms of a sequence $\{a_n\}$ if the first three terms of the sequence are 1, 2, 4.*

5. (Extra Credit). *Let H_n be the nth partial sum of the harmonic series. I.e.* $H_n = \sum_{j=1}^{n} \frac{1}{j}$

   (a) *Prove that $H_{2^n} \geqslant 1 + \frac{n}{2}$*

   (b) *Prove that $H_{2^n} \leqslant 1 + n$*

   (c) *Why does this imply that $H_n \approx \log_2(n)$?*

Let S be a set of $k+1$ horses

$T = S \setminus \{\text{Winnie}\}$ has k horses,
by assumption all horses in T are same color

$W = S \setminus \{\text{Tigger}\}$ has k horses,
by assumption all horses in W have same color

$T \cap W = S \setminus \{\text{Winnie, Tigger}\}$ has $k-1$ horses, all the same color.

$$
\begin{aligned}
\text{color}(\text{Tigger}) &= \text{color}(\text{any horse in } T) \\
&\boxed{= \text{color}(\text{any horse in } T \cap W)} \\
&= \text{color}(\text{any horse in } W) \\
&= \text{color}(\text{Winnie})
\end{aligned}
$$

$(a \leq b \text{ and } b \leq a) \longrightarrow a = b$

S = set of all horses

$X = \mathcal{P}(S)$

$= \{$ subsets of $S \}$

elts. of X are sets of horses

order the elts. of X by

$a \preceq b$ means $|a| \leq |b|$

$W = \{$ Winnie $\}$

$T = \{$ Tigger $\}$

$\rightarrow W \preceq T$

and

$T \preceq W$

but

$W \neq T$

$$H\left(\frac{p}{q}\right) = \max(|p|, |q|)$$

Height	1	2	3	...
#s	0, 1, −1	$\frac{1}{2}, -\frac{1}{2}$, 2, −2	$\pm\frac{1}{3}, \pm\frac{2}{3}$, ±3	...