

1. Find all positive integers n with $\varphi(n) = 4$

2. Do there exist $m, n \in \mathbb{Z}_{>0}$ so that $(m, n) > 1$ and $\varphi(mn) = \varphi(m)\varphi(n)$? What does this allow you to say about the truth of the claim “ $(m, n) = 1$ if and only if $\varphi(mn) = \varphi(m)\varphi(n)$?”

3. For which positive integers n does $\varphi(3n) = 3\varphi(n)$?

4. Let $n \in \mathbb{Z}_{>0}$. Show that there exists $r \geq 1$ so that $\varphi^r(n) = 1$. Here, $\varphi^r(n)$ means the r th iterate of φ , i.e. $\varphi(\varphi(\varphi(\cdots\varphi(n))))$ where φ is composed with itself r times.