1. For positive real numbers a and b with $b > 1$, what does $\log_b(a)$ mean?

2. The goal for this section is to come up with a good definition for $\log_b(a) \bmod m$. We'll start by doing an example. Recall that we have the following table of powers for $(\mathbb{Z}/9\mathbb{Z})^\times$:

x	1	2	3	4	5	6
1^x	1	1	1	1	1	1
2^x	2	4	8	7	5	1
4^x	4	7	1	4	7	1
5^x	5	7	8	4	2	1
7^x	7	4	1	7	4	1
8^x	8	1	8	1	8	1

When working mod 9...

(a) What should $\log_2(2)$ be?

(b) What should $\log_2(4)$ be?

(c) What should $\log_2(8)$ be?

(d) What should $\log_2(7)$ be?

(e) Can you come up with another reasonable answer to the previous question? What about a third answer? A fourth?

(f) Can you give a well-defined function $\log_2(n)$ for all $n \in (\mathbb{Z}/9\mathbb{Z})^\times$? If not, can you give a well-defined function $\log_2(n)$ up to some modulus?

(g) What should $\log_5(1)$ be?

(h) What should $\log_5(5)$ be?

(i) What should $\log_5(7)$ be?

(j) Can you give a well-defined function $\log_5(n)$ for all $n \in (\mathbb{Z}/9\mathbb{Z})^\times$? If not, can you give a well-defined function $\log_5(n)$ up to some modulus?

(k) What should $\log_7(7)$ be?

(l) What should $\log_7(4)$ be?

(m) What should $\log_7(5)$ be?

(n) Can you give a well-defined function $\log_5(n)$ for all $n \in (\mathbb{Z}/9\mathbb{Z})^\times$? If not, can you give a well-defined function $\log_5(n)$ up to some modulus?

3. Here's another example. Recall that we have the following table of powers for $(\mathbb{Z}/8\mathbb{Z})^\times$

x	1	2	3	4
1^x	1	1	1	1
3^x	3	1	3	1
5^x	5	1	5	1
7^x	5	1	5	1

Does there exist a base b so that $\log_b(n)$ is a well-defined function on $(\mathbb{Z}/8\mathbb{Z})^\times$ (up to some modulus)? If yes, give a table of values of $\log_b(n)$ for $n \in (\mathbb{Z}/8\mathbb{Z})^\times$. If no, why not?