Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- Remember to let me know if you can't do your peer review this week (for any reason)!

Here's the schedule of when things are due for this assignment.

| **Component Due** | **Date** |
|---|---|
| Draft of Solutions | Thursday, April 7 |
| Peer Review | Tuesday, April 12 |
| Final Copy | Friday, April 15 |

And finally, here are the problem statements.

1. *Suppose that* $n > 2$ *and* $c_1, \ldots, c_{\varphi(n)}$ *is a reduced residue system modulo* $n$*. Show that*
$$c_1 + c_2 + \cdots + c_{\varphi(n)} \equiv 0 \mod n$$
*Hint: Use the fact that if* $x$ *is relatively prime to* $n$*, so is* $-x$*.*
2. *Suppose that* $a$ *and* $b$ *are relatively prime integers greater than 1. Show that* $a^{\varphi(b)} + b^{\varphi(a)} \equiv 1 \mod ab$
3. *Find all positive integers* $n$ *such that* $\varphi(n) = 12$*. Be sure to prove that you have found all solutions.*
4. *For which positive integers* $n \geqslant 2$ *does* $\varphi(n) \mid n$*?*
*Hint: Use the fundamental theorem of arithmetic to write* $n = p_1^{e_1} \cdots p_g^{e_g}$ *where* $p_1 < p_2 < \cdots < p_g$*. Use the fact that* $\varphi(n) \mid n$ *to find out what* $p_1$ *has to be, then figure out what* $p_2$ *has to be, etc.*
5. *(Extra Credit—and don't use the internet for this one) Prove that* $\lim_{n\to\infty} \varphi(n) = \infty$