

1. Show that 3 is a primitive root modulo 34. Find all primitive roots modulo 34.

2. Show that there are no primitive roots modulo 12.

3. Show that if m is a positive integer and $a \in (\mathbb{Z} / m\mathbb{Z})^\times$ with $\text{ord}_m(a) = m - 1$, then m is prime.

4. Suppose that r and r' are primitive roots modulo n . Show that rr' is not a primitive root modulo n .
Hint: Use the fact that r^k is a primitive root modulo n if and only if k is relatively prime to $\varphi(n)$.

5. Does the expression $\lim_{n \rightarrow \infty} \text{ord}_n(7)$ make sense? Why or why not? If it makes sense, does the limit converge? If yes, what does it converge to?