Some notes on the homework assignment:

- Remember to acknowledge your collaborators and cite your sources. The template has a section built in for you to do this (see the comments in the template).
- Remember to let me know if you can't do your peer review this week (for any reason)!

Here's the schedule of when things are due for this assignment.

Component Due	**Date**
Draft of Solutions	Thursday, April 21
Peer Review	Tuesday, April 26
Final Copy	Friday, April 29

And finally, here are the problem statements.

1. *For a positive integer n, show that*

$$\tau(n^2) = \#\{(a,b) \in \mathbb{Z}^2 : a,b > 0 \text{ and } \operatorname{lcm}(a,b) = n\}$$

Hint: Count the number of elements of the above set by factoring n into primes, say $n = p_1^{e_1} \cdots p_g^{e_g}$. What can you say about the prime factorizations of a and b? How does that relate to the prime factorization of $\operatorname{lcm}(a,b)$?

2. *A partition of n is said to be <u>self-conjugate</u> if it is its own conjugate.*
   (a) *Suppose that $(\lambda_1, \dots, \lambda_r)$ is a partition of n. Let $S = \{\lambda_j : \lambda_j \geqslant j\}$. Show that if $\lambda_k \in S$, then $\lambda_{k-1} \in S$. Conclude that S has the form $\{\lambda_1, \dots, \lambda_t\}$ for some t.*
   (b) *Let $(\lambda_1, \dots, \lambda_r)$ be a self-conjugate partition of n. Suppose that a dot in the Ferrers diagram of $(\lambda_1, \dots, \lambda_r)$ is in row j and column ℓ. Show that either $\lambda_j \geqslant j$ or $\lambda_\ell \geqslant \ell$.*
   (c) *Let $(\lambda_1, \dots, \lambda_r)$ be a self-conjugate partition of n. Let $k = \#\{j : \lambda_j \geqslant j\}$. For each $1 \leqslant i \leqslant k$, define $\rho_i = 2\lambda_i - (2i-1)$. Prove that $(\rho_1, \dots, \rho_k)$ is a partition of n with distinct, odd parts.*
   Hint: ρ_1 counts the number of dots in either the first row or column of the Ferrers diagram of $(\lambda_1, \dots, \lambda_r)$. What do $\rho_2, \dots, \rho_k$ represent?
   (d) *Let O be the set of odd positive integers. Show that the number of self-conjugate partitions of n is equal to $p_O^D(n)$.*
3. *Use the previous problem to show that $p(n)$ is odd if and only if $p_O^D(n)$ is odd.*
4. *(Extra Credit) Use Ferrers diagrams to show that $p^D(n)$ is equal to the number of partitions of the form $(\lambda_1, \dots, \lambda_k)$ where for each i so that $1 \leqslant i \leqslant \lambda_1$, there exists a j so that $i = \lambda_j$.*