Math 348 | # Homework 6 | Spring 2022

1. *Suppose that $m > 2$ has a primitive root. Show that the product of all elements of $(\mathbb{Z}\,/\,m\mathbb{Z})^{\times}$ is congruent to -1 mod m.*

   Your answer here...

2. *Suppose that m has a primitive root, r. Show that* $a \equiv b \mod m$ *if and only if* $\operatorname{ind}_r(a) \equiv \operatorname{ind}_r(b) \mod \phi(m)$.
   Note: This shows that "taking indices is invertible."

   Your answer here...

3. *For which positive integers a is the congruence* $ax^4 \equiv 2 \mod 13$ *solvable?*

Your answer here...

4. *Let* $N = 2^j u$ *be a positive integer where* $j \geq 0$ *and* u *is odd. Let* p *be an odd prime and factor* $p - 1 = 2^s t$ *where* s *and* t *are positive integers with* t *odd. Show that if* $0 \leq j < s$, *then there are* $2^j(t, u)$ *incongruent solutions of* $x^N \equiv -1 \mod p$. *Show that there are no solutions otherwise.*

   Your answer here...