Ex 1 Find the elementary matrix which corresponds to:

(a) swapping the first and third row of a 3×3 matrix.
(b) swapping the first and third row of a 3×4 matrix.
(c) swapping the first and third row of a 4×3 matrix.
(d) swapping the ith and jth row of an $m \times n$ matrix.
(e) adding twice the first row to the third row of a 3×3 matrix.
(f) adding twice the first row to the third row of a 3×4 matrix.
(g) adding c times the ith row to the jth row of an $m \times n$ matrix.
(h) multiplying the second row of a 3×3 matrix by -7.
(i) multiplying the ith row of an $m \times n$ matrix by c.

Ex 2 Let E be the elementary matrix which corresponds to swapping the first and second row of a 3×3 matrix. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} -2 & 4 & 6 & -3 \\ 0 & 2 & 1 & -1 \\ -5 & 2 & \pi & 0 \end{pmatrix}$$

(a) What is EA?
(b) What is AE?
(c) For any matrix M, what does multiplying by E on the left do? What does multiplying by E on the right do?
(d) What is EB? What is BE? Does this line up with your answer to the previous part?

Ex 3 Explain why the matrix inversion algorithm works.

Ex 4 Compute the determinant of the matrix

$$\begin{pmatrix} -2 & 3 & 1 \\ 0 & -1 & -3 \\ 4 & 2 & 5 \end{pmatrix}.$$

Ex 5 Let $A = \left(\begin{smallmatrix} 1 & 2 \\ 3 & 4 \end{smallmatrix}\right)$.

(a) Compute all four minors of A.
(b) Compute all four cofactors of A.
(c) Compute the determinant of A using each of the four different cofactor expansions.

Ex 6 Compute the determinant of the matrix

$$\begin{pmatrix} -1 & 4 & 3 & 0 & 0 & 3 \\ -8 & 2 & 0 & 2 & 0 & 1 \\ 3 & -1 & 4 & 2 & 2 & 9 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & -9 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & -2 \end{pmatrix}$$

Hint: Make good choices when choosing the rows/columns along which to do the cofactor expansion.

Ex 7 Suppose you are given three numbers a, b, and d. Is it always possible to choose a number c so that the matrix $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ has $\det(A) = 0$?

Ex 8 Give an example of a 2×2 matrix A so that:

(a) $\det(A) = 5$.
(b) $\det(A^{-1}) = 5$.
(c) for some other 2×2 matrix B, $\det(A + B) \neq \det(A) + \det(B)$.
(d) for every other 2×2 matrix B, $\det(A + B) = \det(A) + \det(B)$.