Ex 1 Which of the following are matrices?

(a)
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 0 & 2 & -1 \\ 1 & \pi & 7 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 0 & 2 & -1 \\ 1 & \pi & \end{pmatrix}$$

(d)
$$\begin{aligned} 2x + 3y &= 1 \\ 3x - 4y &= 2 \end{aligned}$$

(e)
$$\begin{pmatrix} 0 \end{pmatrix}$$

Ex 2 What is the rank of matrix A?

(a)
$$A = \begin{pmatrix} 1 & 0 & 5 & 11 \\ 0 & 1 & 9 & -1 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

(b)
$$A = \begin{pmatrix} 2 & 3 & 5 & 7 \\ 2020 & 2021 & 2022 & 2023 \end{pmatrix}$$

(c)
$$A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 7 & -3 \\ 2 & 9 & -4 \\ 2 & 6 & 11 \end{pmatrix}$$

Ex 3 True or False? The rank of an $m \times n$ matrix must be less than or equal to $\min(m, n)$. Justify your answer.

Ex 4 How many solutions are there to the following systems of linear equations? Answer this question using your answers to Ex 2.

(a)
$$\begin{aligned} x + 5z &= 11 \\ y + 9z &= -1 \\ z &= 4 \end{aligned}$$

(b)
$$\begin{aligned} 2x + 3y + 5z &= 7 \\ 2020x + 2021y + 2022z &= 2023 \end{aligned}$$

(c)
$$\begin{aligned} x + 3y &= 2 \\ 4x + 7y &= -3 \\ 2x + 9y &= -4 \\ 2x + 6y &= 11 \end{aligned}$$

Ex 5 Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$, and $C = \begin{pmatrix} -4 & 7 & 1 \\ 2 & -3 & -5 \end{pmatrix}$. Compute the following or explain why such a computation does not make sense.

(a) $2A$
(b) $A + B$
(c) $A + C$
(d) $B + 2$
(e) $B + 2I_2$
(f) $3A - B$
(g) AB
(h) BA
(i) AC
(j) CA
(k) A^2
(l) C^2
(m) $A^2B - 2A + B$
(n) A^T
(o) C^T
(p) $A^\mathsf{T}B^\mathsf{T}$
(q) $(AB)^\mathsf{T}$

Ex 6 What is the relation between the matrices in the last two parts of the previous question?