Matrix Operations

Ex 1 Suppose that A is an $n \times n$ matrix, B is an $m \times n$ matrix, and C is an $n \times p$ matrix. Compute the dimensions of the following matrices OR explain why such an operation does not make sense.

(a) BA

(b) AB

(c) BA^{T}

(d) $(BA)^{\mathsf{T}}$

(e) $2B^{\mathsf{T}}$

(f) $((AC)^{\mathsf{T}} - BI_n)^{\mathsf{T}}$

Ex 2 Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$
$$B = \begin{pmatrix} -1 & 2 & 4 \\ 2 & -3 & 0 \\ 4 & 0 & 1 \end{pmatrix}$$

(a) True or false? A and B are both symmetric matrices.

(b) Compute AB

(c) Compute BA

(d) True or false? If S and T are symmetric $n \times n$ matrices, then ST is also a symmetric $n \times n$ matrix.

(e) True or false? If S and T are symmetric $n \times n$ matrices, then ST must equal TS.

(f) True or false? If S and T are symmetric $n \times n$ matrices, then ST must equal $(TS)^{\mathsf{T}}$.

Ex 3 Use the matrix inversion algorithm to compute the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

OR explain why the inverse of A does not exist.

Ex 4 Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -3 & -2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$
$$B = \begin{pmatrix} -1 & -3 & -4 \\ 0 & 0 & 8 \\ 3 & 1 & -4 \end{pmatrix}$$
$$C = \begin{pmatrix} -1/8 & -3/8 & -1/2 \\ 0 & 0 & 1 \\ 3/8 & 1/8 & -1/2 \end{pmatrix}$$

(a) Is B the inverse matrix of A? Answer this question WITHOUT computing the inverse matrix of A.

(b) Is C the inverse matrix of A? Answer this question WITHOUT computing the inverse matrix of A.

(c) What is the relationship between matrices B and C?

(d) What is CB^{-1}? Answer this question WITHOUT computing the inverse of B.