Ex 1 You have a language consisting of only three words: "every," "student," and "learns." You also have a large book filled with sentences consisting of those three words. After doing some statistical analysis on the book, you find that:

- If you find the word "every," the next word is "every" 10% of the time, the next word is "student" 60% of the time, and the next word is "learns" 30% of the time.

- If you find the word "student," the next word is "every" 30% of the time, the next word is "student" 20% of the time, and the next word is "learns" 50% of the time.

- If you find the word "learns," the next word is "every" 50% of the time, the next word is "student" 40% of the time, and the next word is "learns" 10% of the time.

The first word in the book is "Every." What is the probability that...

(a) ...the word "learns" appears next?

(b) ...the word "learns" appears 2 words after the word "every?"

(c) ...the word "learns" appears n words after the word "every?"

Ex 2 There are three towns: A, B, and C. Each year, 10% of town A's residents move to town B and 20% of town A's residents move to town C. Each year, 40% of town B's residents move to town A and 10% move to town C. Each year, 20% of town C's residents move to town A and 20% move to town B. Initially, town A has 300 people, town B has 100 people, and town C has 200 people. How many people live in each town after a long time?

Ex 3 Suppose that M is a Markov matrix with positive entries. Suppose that M has 1-eigenvectors of the form

$$t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \text{for } t \neq 0.$$

Suppose that

$$v = \begin{pmatrix} 1 \\ 3 \\ 9 \end{pmatrix}$$

is an initial condition. What does $M^n v$ approach as n grows?

Ex 4 You are battling some Pokémon against your friend. In a battle, you each typically bring 6 Pokémon, then you battle, and you see who wins. You are better than your friend, so to make things more competitive, if you win a match, you bring one fewer Pokémon to the next battle, down to a minimum of four. Your probabilities of winning are as follows:

Number of Pokémon you bring	Probability you win
6	0.8
5	0.6
4	0.3

What proportion of games do you win in the long run?