Computational Questions

Ex 1 Diagonalize the matrix

$$\begin{pmatrix} -2 & 2 & 1 \\ 0 & 3 & -1 \\ 0 & 6 & -2 \end{pmatrix}$$

in two different ways, if possible.

Ex 2 Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}.$$

Find a formula for A^k (in terms of k).

Ex 3 Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Find a formula for A^k (in terms of k).

Conceptual Questions

Ex 1 Classify each of the following statements as "always true," "sometimes true," or "never true."

(a) Suppose that A is diagonalizable, but not diagonal. Then there are multiple diagonalizations of A. (Recall that the diagonalization of A is the matrix D in the relation $A = PDP^{-1}$. I'm not asking if there are multiple matrices P. I'm asking if there are multiple matrices D.)

(b) Suppose that A is a 3×3 matrix with only two distinct eigenvalues. Then A is not diagonalizable.

(c) Every matrix is diagonalizable if you allow complex eigenvalues and eigenvectors.

Ex 2 Suppose that A is a Jordan block, that is, A is an $n \times n$ matrix of the form

$$A = \begin{pmatrix} \lambda & 1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & 0 & \cdots & 0 \\ 0 & 0 & \lambda & \ddots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda & 1 \\ 0 & 0 & 0 & \cdots & 0 & \lambda \end{pmatrix}$$

for some $\lambda \in \mathbb{R}$. For which values of n is A diagonalizable?