Cross Products and Linear Maps Math 211, Fall, 2023

Ex 1 Suppose that $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$ and

$$\vec{u} \times \vec{v} = \begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}, \qquad \vec{u} \times \vec{w} = \begin{pmatrix} 16 \\ 6 \\ -10 \end{pmatrix}, \qquad \text{and } \vec{v} \cdot \vec{w} = -8.$$

Find each of the following quantities OR explain why such a task is nonsensical or impossible.

(a) $(\vec{u} \times \vec{v}) \times (\vec{u} \times \vec{w})$.

(b) $\vec{u} \cdot (\vec{u} \times \vec{v})$

(c) The angle between $\vec{u}$ and $\vec{v}$.

(d) $(\vec{v} + \vec{w}) \times \vec{u}$

Ex 2 (a) Find the equation of the line passing through the points $(0, 1, 0)$ and $(-2, 3, 1)$.

(b) Find the equation of the line passing through the points $(0, 1, 0)$ and $(4, 1, 7)$.

(c) Find the equation of the plane containing the points $(0, 1, 0)$, $(-2, 3, 1)$, and $(4, 1, 7)$.

Ex 3 Find the distance between the lines

$$L_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$L_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

WITHOUT using the same method that Gilad did in class. Hint: use the cross-product!

Ex 4 Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects a point across the line $y = -3x$. Is T a linear map? If yes, what is the matrix of T? If no, why not?

Ex 5 Consider the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects a point across the line $y = -3x + 1$. Is T a linear map? If yes, what is the matrix of T? If no, why not?

Ex 6 Consider the function $T : \mathbb{R} \to \mathbb{R}$ which maps an input x to $-3x + 1$. Is T a linear map? If yes, what is the matrix of T? If no, why not?

Ex 7 Consider the function $T : \mathbb{R}^3 \to \mathbb{R}^3$ which maps input $\vec{x}$ to $\vec{x} \times \vec{x}$. Is T a linear map? If yes, what is the matrix of T? If no, why not?

Ex 8 Consider the function $T : \mathbb{R}^3 \to \mathbb{R}^3$ which maps input $\vec{x}$ to

$$\vec{x} \times \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}.$$

Is T a linear map? If yes, what is the matrix of T? If no, why not?

Ex 9 Suppose that $n \geqslant 1$ and consider the function $T : \mathbb{R}^n \to \mathbb{R}$ which maps input $\vec{x}$ to $\vec{x} \cdot \vec{x}$. Is T a linear map? If yes, what is the matrix of T? If no, why not?