Computational Questions

Ex 1 Find the eigenvectors and eigenvalues of

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

Ex 2 What values of x make the matrix $A = \left(\begin{smallmatrix} 2 & x \\ 3 & -1 \end{smallmatrix}\right)$ have eigenvalue $\lambda = 1$? Alternatively, explain why no value of x makes A have the eigenvalue $\lambda = 1$.

Ex 3 What values of x make the matrix $A = \left(\begin{smallmatrix} 2 & x \\ 0 & -1 \end{smallmatrix}\right)$ have eigenvalue $\lambda = 1$? Alternatively, explain why no value of x makes A have the eigenvalue $\lambda = 1$.

Ex 4 What values of x make the matrix $A = \left(\begin{smallmatrix} 2 & x \\ 0 & 1 \end{smallmatrix}\right)$ have eigenvalue $\lambda = 1$? Alternatively, explain why no value of x makes A have the eigenvalue $\lambda = 1$.

Conceptual Questions

Ex 1 What are the eigenvectors of a diagonal matrix whose diagonal entries are all distinct?

Ex 2 An upper triangular matrix is a square matrix whose entries are 0 if the row index is greater than the column index (but the rest of the entries can be anything). What are the eigenvalues of an upper triangular matrix?

Ex 3 Classify each of the following statements as “always true,” “sometimes true,” or “never true.”

(a) An $n \times n$ matrix with real entries has n distinct (real) eigenvalues.

(b) An $n \times n$ matrix with real entries has n (real) eigenvalues including multiplicity (e.g. an eigenvalue with multiplicity 2 counts as 2 eigenvalues here).

(c) Suppose that A and B are similar matrices. Then A and B have the same eigenvectors.

Ex 4 Construct a matrix A with the following properties:

(a) A has eigenvalue $\lambda_1 = 3$ with multiplicity 2.

(b) A has eigenvalue $\lambda_2 = -1$ with multiplicity 1.

(c) A has one basic eigenvector for $\lambda_1 = 3$.

(d) A has one basic eigenvector for $\lambda_2 = -1$