Lecture Examples

Ex 1 In a newspaper poll concerning violence on television, 600 people were asked "Is there too much violence on television?" Their responses were

	Yes	No	Don't Know	Total
Men	162	95	23	280
Women	256	45	19	320
Total	418	140	42	600

(a) Find the probability that a person responded "yes."

$Y = \{x \mid x \text{ responded "yes"}\}$

$$p(Y) = \frac{418}{600} \approx .70$$

(b) Find the probability that a man responded "yes" (that is, find the probability that a person responded yes, given that the person is a man). Rephrased, find the probability that if a person is a man, then they responded yes.

$M = \{x \mid x \text{ is a man}\}$

$$p(Y|M) = \frac{n(Y \cap M)}{n(M)} = \frac{162}{280} \approx .58$$

(c) Find the probability that a response is from a man, given that the response is "yes." Rephrased, find the probability that if a person answered yes, they are a man.

$$p(M|Y) = \frac{n(M \cap Y)}{n(Y)} = \frac{162}{418} \approx .39$$

(d) Find the probability that a response is yes and is from a man.

$$p(Y \cap M) = \frac{n(Y \cap M)}{n(S)} = \frac{162}{600} \approx .27$$

Ex 2 A single card is drawn from a 52-card deck.

(a) What is the probability that the card is a queen?

$Q = \{x \mid x \text{ is a queen}\}$

$$p(Q) = \frac{n(Q)}{n(S)} = \frac{4}{52}$$

(b) What is the probability that the card is a queen, given that it is a face card?

$F = \{x \mid x \text{ is a face card}\}$

$$p(Q|F) = \frac{n(Q \cap F)}{n(F)} = \frac{4}{12}$$

(c) What is the probability that the card is a spade?

$P = \{x \mid x \text{ is a spade}\}$

$$p(P) = \frac{13}{52}$$

(d) What is the probability that the card is a spade, given that it is black?

$B = \{x \mid x \text{ is black}\}$

$$p(P|B) = \frac{n(P \cap B)}{n(B)} = \frac{13}{26}$$

Ex 3 Two cards are dealt from a 52-card deck. Find the probability that both cards are kings using...

(a) ...conditional probability.

$K_1 = \{(x,y) \mid x \text{ is a king}\}$

$K_2 = \{(x,y) \mid y \text{ is a king}\}$

$$P(K_1 \cap K_2) = P(K_1)\,P(K_2 \mid K_1) = \frac{4}{52} \cdot \frac{3}{51} = .005$$

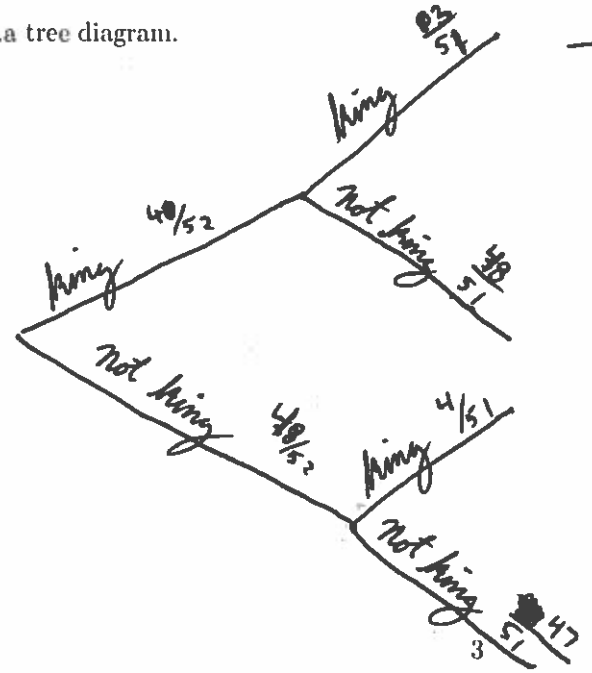
(b) ...combinatorics.

$$n(K_1 \cap K_2) = {}_4C_2 = 6$$

$$n(S) = {}_{52}C_2 = 1326$$

$$P(K_1 \cap K_2) = \frac{6}{1326} = .005$$

(c) ...a tree diagram.



$$\frac{4}{52} \cdot \frac{3}{51} = .005$$

Ex 4 A single six-sided die is rolled twice.

(a) Find the probability of rolling a 4 on the second die if the first roll was a 2. Compare this to the probability of rolling a 4 on the second die.

$E = \{(x,y) \mid y = 4\}$ $F = \{(x,y) \mid x = 2\}$

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{1}{6}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$P(E) = P(E \mid F)$

(b) Find the probability of rolling a sum of 6 if the first roll was 2. Compare this the the probability of rolling a sum of 6.

$G = \{(x,y) \mid x + y = 6\}$

$$P(G) = \frac{n(G)}{n(S)} = \frac{5}{36} \approx .14$$

$$P(G \mid F) = \frac{n(G \cap F)}{n(F)} = \frac{1}{6} \approx .17$$

$P(G) \neq P(G \mid F)$

Ex 5 You roll two six-sided dice. Consider the two events: E: you roll an odd sum. F: you roll a sum of 4. Are the two events mutually exclusive or not? Independent or dependent?

E and F are mutually exclusive

$P(E) = .5$

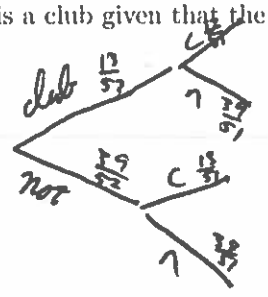
$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{0}{6} = 0$$

E and F are not independent

Ex 6 Two cards are drawn from a standard deck of 52-cards. Find the probability of the second card being a club. Compare this to the probability that the second card is a club given that the first card was a club.

$X = \{(x,y) \mid x \text{ is a club}\}$

$Y = \{(x,y) \mid y \text{ is a club}\}$



$$P(Y) = \left(\frac{13}{52}\right)\left(\frac{12}{51}\right) + \left(\frac{39}{52}\right)\left(\frac{13}{51}\right)$$

$$= \frac{13}{52}$$

$$P(Y \mid X) = \frac{12}{51}$$

On-Your-Own Examples

Ex 1 The following is a table depicting the 2008 U.S. population in thousands.

Age (years)	0 - 4	5 - 19	20 - 44	45 - 64	65 - 84	85 +	Total
Male	10,748	31,549	53,060	38,103	14,601	1,864	149,925
Female	10,258	30,085	51,432	39,955	18,547	3,858	154,135
Total	21,006	61,634	104,492	78,058	33,148	5722	304,060

(a) Find the probability that a person in the U.S. in 2008 was age 20 - 44.

$$\frac{104492}{304060} \approx .34$$

(b) Find the probability that a person in the U.S. in 2008 was female, given that the person was age 20 - 44.

$$\frac{51432}{104492} \approx .49$$

(c) Find the probability that a person was female and age 20 - 44.

$$\frac{51432}{304060} \approx .17$$

(d) Find the probability that a person was male and age 5 - 19 or 65 - 84.

$$\frac{31549 + 14601}{304060} \approx .15$$

(e) Find the probability that a person was age 45 - 64 given that the person was male.

$$\frac{38103}{149925} \approx .25$$

Ex 2 Two six-sided dice are rolled.

(a) What is the probability that the sum is 8?

$E = \{(x,y) \mid x+y = 8\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

(b) What is the probability that the sum is 8 given that one die rolled a 5?

$F = \{(x,y) \mid x = 5\}$ $G = \{(x,y) \mid y = 5\}$

$$P(E \mid F \cup G) = \frac{n(E \cap (F \cup G))}{n(F \cup G)} = \frac{2}{n(F)+n(G)-n(F \cap G)} = \frac{2}{6+6-1} = \frac{2}{11} \approx .18$$

(c) What is the probability that the sum is 8 given that one die rolled a 1?

If one die is a 1, the sum can't be 8, so the probability is 0.

(d) What is the probability that one die came up 5, given that the sum is 8?

$$P(F \cup G \mid E) = \frac{n(E \cap (F \cup G))}{n(E)} = \frac{2}{5} = .4$$

Ex 3 A math teacher gave her class two tests. If 25% of the class passed both tests and 42% of the class passed the first test, what fraction of those who passed the first test also passed the second test?

$F = \{x \mid x \text{ passed the first test}\}$

$T = \{x \mid x \text{ passed the second test}\}$

$$P(T \mid F) = \frac{P(T \cap F)}{P(F)} = \frac{.25}{.42} \approx .60$$

Ex 4 At Kennedy Middle School the probability that a student takes Technology and Spanish is 8.7%. The probability that a student takes Technology is 68%.

(a) What is the probability that a student takes Spanish given that the student is taking Technology?

$P = \{x \mid x \text{ takes Spanish}\}$ $T = \{x \mid x \text{ takes technology}\}$

$$p(P \mid T) = \frac{p(P \cap T)}{p(T)} = \frac{.087}{.68} \approx .13$$

(b) If the probability of a student taking Spanish is 46%, find the probability that a student takes Technology given that the student is taking Spanish.

$$p(T \mid P) = \frac{p(T \cap P)}{p(P)} = \frac{.087}{.46} \approx .19$$

6-sided

Ex 5 A pair of dice are rolled. Find the probability of each of the events:

(a) The sum is 6.

$E = \{(x,y) \mid x+y = 6\}$

$$p(E) = \frac{5}{36}$$

(b) The sum is 6 given that the sum is even.

$F = \{(x,y) \mid x+y \text{ is even}\}$

$$p(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{5}{18}$$

(c) The sum is 6 given that the sum is odd.

$$p(E \mid F') = \frac{n(E \cap F')}{n(F')} = 0$$

(d) The sum is even given that the sum is 6.

$$p(F \mid E) = \frac{n(E \cap F)}{n(E)} = \frac{5}{5} = 1$$

Ex 6 Two cards are dealt from a standard 52-card deck. - do (e) first!

(a) Find the probability that the first card is a club.

$$\frac{13}{52}$$

(b) Find the probability that second card is a club given that the first card was a club.

$$\frac{12}{51}$$

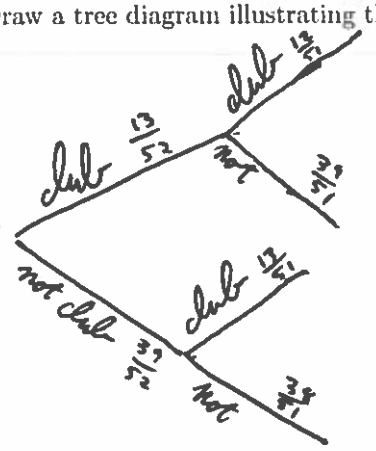
(c) Find the probability that the first and second cards are both clubs.

$$\left(\frac{13}{52}\right)\left(\frac{12}{51}\right) = .06$$

(d) Find the probability that exactly one of the two cards is a club.

$$\left(\frac{13}{52}\right)\left(\frac{39}{51}\right) + \left(\frac{39}{52}\right)\left(\frac{13}{51}\right) \approx .38$$

(e) Draw a tree diagram illustrating the above.



Ex 7 A card is drawn from a standard 52-card deck. Let E be the event that the card is a king. Let F be the event that the card is face card.

(a) Are the events E and F mutually exclusive?

They are not mutually exclusive because $E \cap F$ is nonempty (it has the king of hearts, for example)

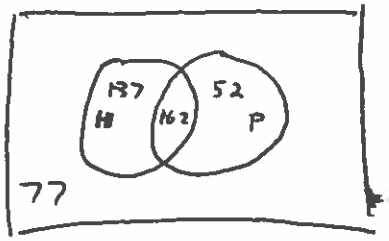
(b) Are the events E and F independent?

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} = \frac{4}{12} \approx .33$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} \approx .08$$

The events are not independent

Ex 8 A department store surveyed 428 shoppers. According to the survey, 214 made a purchase, 299 were satisfied with the service, and 52 of those who made a purchase were not satisfied with the service. Let H be the event that a shopper was happy with the service and let P be the event that a shopper made a purchase.



(a) Are the events H and P independent?

$$P(H \mid P) = \frac{n(H \cap P)}{n(P)} = \frac{162}{214} \approx .76$$

$$P(H) = \frac{137}{428} \approx .32$$

H and P are not independent

(b) Are the events H' and P' independent?

$$P(H' \mid P') = \frac{n(H' \cap P')}{n(P')} = \frac{77}{77+137} \approx .36$$

$$P(H') = \frac{52+77}{428} \approx .30$$

H' and P' are not independent