Section 3.5 Lecture Guide

Math 105, Summer 2019

Lecture Examples

Ex 1 What is the expected value of rolling a six-sided die if the number of points awarded is twice what you rolled (e.g. you get 2 points for rolling a 1, 4 points for rolling a 2, etc.)?

outcome	1	2	3	4	5	6
probability	1/6	1/6	1/6	1/6	1/6	1/6
value	2	4	6	8	10	12

$$EV: \left(\frac{1}{6}\right)\cdot 2 + \left(\frac{1}{6}\right)\cdot 4 + \left(\frac{1}{6}\right)\cdot 6 + \left(\frac{1}{6}\right)\cdot 8 + \left(\frac{1}{6}\right)\cdot 10 + \left(\frac{1}{6}\right)\cdot 12 = 7$$

Ex 2 A game is played in which you bet \$1 on a number between 1 and 100. If your number matches the computer's randomly chosen number, you win \$50. If you're wrong, you lose the dollar you bet. Is this game worth playing?

outcome	win	lose
probability	1/100	99/100
value	50	-1

$$EV: \left(\frac{1}{100}\right)\cdot 50 + \left(\frac{99}{100}\right)\cdot(-1) = -.49$$

The game is not worth playing

Ex 3 You run a store and find that your possible profits for next month have the following probability distribution:

Profit	-\$10	\$0	\$10	\$20
Probability	.3	.5	.15	.05

What is the expected value of your profit next month?

$$EV: (.3)(-10) + (.5)(0) + (.15)(10) + (.05)(20)$$
$$= -.5$$

Ex 4 On a multiple choice test with 5 possible answers for each question (a) through (e), you get 1 point for each correct answer and lose $\frac{1}{2}$ point for each incorrect answer. Find the expected value of a random guess.

outcome	correct	incorrect
probability	$1/5$	$4/5$
value	1	$-1/2$

$$EV: \left(\tfrac{1}{5}\right)(1) + \left(\tfrac{4}{5}\right)\left(-\tfrac{1}{2}\right) = -.2$$

On-Your-Own Examples

Ex 1 You have been offered a chance to play a dice game where you roll a single six-sided die and if you roll a 1, 2, 3, or 6, you win \$50, but if you roll a 4 or 5, you lose \$70. Find the expected value of the game. Is it worth playing?

outcome	1	2	3	4	5	6
probability	1/6	1/6	1/6	1/6	1/6	1/6
value	50	50	50	-70	-70	50

$$EV = (1/6)(50) + (1/6)(50) + (1/6)(50) + (1/6)(-70) + (1/6)(-70) + (1/6)(50)$$
$$= 10.$$
The game is worth playing

Ex 2 A dice game requires a \$1 bet to play. If you roll two six-sided dice and the sum of the dice is 11 or more, you keep your dollar and win an additional \$10. For any other roll, you lose your dollar. Is it worth playing this game?

event	11+	other
probability	3/36	33/36
value	10	-1

$$E.V. = (3/36)(10) + (33/36)(-1) \approx -.08$$
The game is not worth playing

Ex 3 Consider the following game: three coins are flipped. If all three coins match, you win \$10; if exactly two heads are flipped, you lose \$4; and if exactly two tails are flipped, you lose \$3. What is the expected value of playing this game?

$$S = \{(H,H,H), (H,H,T), (H,T,H), (T,H,H), (H,T,T), (T,H,T), (T,T,H), (T,T,T)\}$$

event	match	2 H	2 T
probability	2/8	3/8	3/8
value	10	-4	-3

$$E.V. = (2/8)(10) + (3/8)(-4) + (3/8)(-3) = -.125$$

Ex 4 Find the expected value of buying a ticket for a 6/41 lottery if the ticket costs $1, the first prize is worth $1 million, and the second prize is worth $10,000.

event	1^{st}	2^{nd}	lose
probability	$1/{}_{41}C_6 = .0000002$	$\frac{{}_6C_5 \cdot {}_{35}C_1}{{}_{41}C_6} = .00005$	$1 - .0000002 - .00005 = .9999498$
value	1,000,000	10,000	-1

$$E.V. = (.0000002)(1000000) + (.00005)(10{,}000) + (.9999498)(-1)$$
$$= -.2999498$$

* **Ex 5** In the 6/41 lottery described above, assume the second prize remains $10,000. How much would the first prize need to be in order for it to be worth playing the lottery?

event	1^{st}	2^{nd}	lose	
probability	.0000002	.00005	.9999498	
value	V	10,000	-1	

When is expected value 0?

$$0 = .0000002V + (.00005)(10000) + (.9999498)(-1)$$
$$= .0000002V - .4999498$$
$$\Rightarrow .4999498 = .0000002V$$
$$\Rightarrow 2499749 = V$$

First prize must be at least $2,499,749