

Lecture Examples

Ex 1 Make a truth table with the statements $p \rightarrow q$ (a conditional), $q \rightarrow p$ (the inverse of the conditional), $(\sim p) \rightarrow (\sim q)$ (the converse of the conditional), and $(\sim q) \rightarrow (\sim p)$ (the contrapositive of the conditional). Which of the statements are equivalent?

Ex 2 Consider the conditional statement “Being an athlete is necessary for being a dancer.” Express the contrapositive...

(a) ...as a sufficient condition.

(b) ...as a necessary condition.

On-Your-Own Examples Summary of equivalences

$\sim(\sim p)$	\equiv	p	negation of a negation
$\sim(p \wedge q)$	\equiv	$(\sim p) \vee (\sim q)$	negation of a conjunction
$\sim(p \vee q)$	\equiv	$(\sim p) \wedge (\sim q)$	negation of a disjunction
$\sim(p \rightarrow q)$	\equiv	$p \wedge (\sim q)$	negation of a conditional
$p \rightarrow q$	\equiv	$(\sim q) \rightarrow (\sim p)$	a conditional and its contrapositive
$q \rightarrow p$	\equiv	$(\sim p) \rightarrow (\sim q)$	the converse and inverse of a conditional
$p \leftrightarrow q$	\equiv	$(p \rightarrow q) \wedge (q \rightarrow p)$	biconditional

Ex 1 Given the following statements, write the natural language sentence represented by the symbols.

p : I am sleeping.

q : I am snoring.

(a) $p \rightarrow q$

(b) $q \rightarrow p$

(c) $(\sim p) \rightarrow (\sim q)$

(d) $(\sim q) \rightarrow (\sim p)$

(e) $p \leftrightarrow q$

Ex 2 Write an equivalent variation of the given conditional.

- (a) If it is not snowing, then it is warm outside.

- (b) You are not a vegetarian if you eat meat.

Ex 3 State the converse of the sentence: If she goes to the store, she will buy ice cream.

Ex 4 Translate the two sentences into symbolic form and use a truth table to determine whether the statements are equivalent.

- (a) Being an automobile that is American-made is sufficient for an automobile having hardware that is not metric.
- (b) Being an automobile that is not American-made is necessary for an automobile having hardware that is metric.