

Exam 2

Math 105, Summer 2019

Name: _____

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 121 points on this exam. That means you should budget about 0.4 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder. There are to be no devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

1. Vocabulary

(a) What is a set?

A set is a collection of objects

(b) What is the cardinality (or cardinal number) of a set?

The cardinality of a set is the number of elements that it has

(c) What is the intersection of two sets, say X and Y ? Don't give the notation, give the definition.

The intersection of X and Y is the set of all elements which are in both X and Y

(d) State De Morgan's Laws for sets.

$$(A \cap B)' = A' \cup B'$$

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(e) What is a combination?

A combination is an outcome when drawing is done without replacement and order doesn't matter

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2. **True or False?** For each question, circle either the word "true" or the word "false." Use the universe U being the set of all whole numbers and sets $X = \{1, 3, 5\}$, $Y = \{2, 3, 5\}$, and $Z = \{n \mid n \text{ is an even whole number and } -6 \leq n \leq 1540\}$

(a) True or False? $5 \in X$

(b) True or False? $5 \in X \cap Y'$

(c) True or False? $X \cup Z = \emptyset$

(d) True or False? $Z' \cap Y = \{3\}$

(e) True or False? $Y \subseteq Z$

3. Use the universe $U = \{1, 2, 3, 4, 5, 6\}$

(a) Give an example of sets X and Y so that $X \cap Y = Y$

$$X = \{1, 2\}$$

$$Y = \{1\}$$

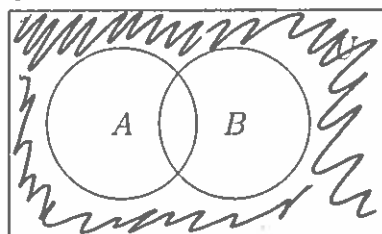
(b) Give an example of sets X and Y so that $n(X \cup Y) = n(X) + n(Y)$

$$X = \{1\}$$

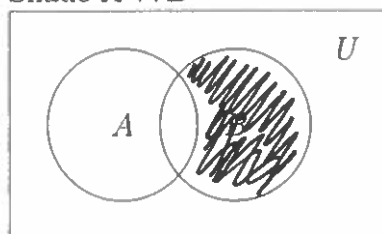
$$Y = \{2\}$$

4. Given the following Venn Diagram, shade the specified set.

(a) Shade $A' \cap B'$



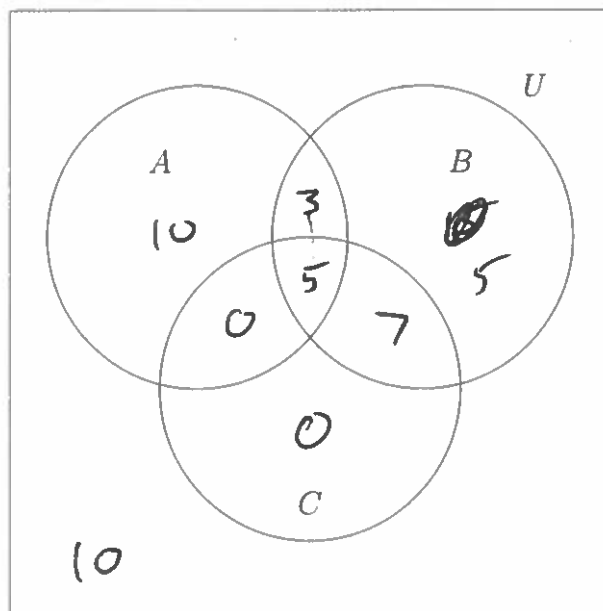
(b) Shade $A' \cap B$



5. You have a universe, U , and sets A , B , and C . You have the following information

$$\begin{array}{ll} n(A) = 18 & n(A \cap B \cap C) = 5 \\ n(B) = 20 & n(A \cap B) = 8 \\ n(C) = 12 & n(B \cap C) = 12 \\ n(U) = 40 & n(A \cap C) = 5 \end{array}$$

Fill out the following Venn Diagram, illustrating the composition of the various sets.



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6. You are building a computer, and you have three remaining choices. You need to purchase a processor, a GPU, and a monitor. You are considering 5 processors, 3 GPUs, and 10 monitors. How many different computers could you build with these options?

$$\frac{5}{p} \cdot \frac{3}{G} \cdot \frac{10}{m} = 150 \text{ computers}$$

7. You are spending a day in Portland and you have time to visit four different places. You are choosing between Powell's, Salt and Straw, Voodoo Donuts, the zoo, OMSI, the rose garden, the Chinese garden, and the market. How many different trips can you take, assuming that going to Powell's and then the zoo is different from going to the zoo and then Powell's?

- drawing without replacement
- order matters

$$8P_4 = 1680 \text{ different trips}$$

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8. Recall that a standard deck of cards has 52 cards: 4 different suits, called hearts, spades, clubs, and diamonds, with each suit having 1 of each of the denominations, 2-10, jack, queen, king, and ace. In how many ways can you be dealt a 5 card hand with...

- (a) ...exactly 2 kings and exactly 3 queens?

$$\frac{{}_4C_2}{{}_K} \cdot \frac{{}_4C_3}{{}_Q} = 24$$

- (b) ...exactly 2 kings?

$$\frac{{}_4C_2}{{}_K} \cdot \frac{{}_{49}C_3}{\text{not } K} = 103,776$$

- (c) ...exactly 3 queens?

$$\frac{{}_4C_3}{{}_Q} \cdot \frac{{}_{48}C_2}{\text{not } Q} = 4512$$

- (d) ...exactly 2 kings or exactly 3 queens?

$$K = \{x \mid x \text{ is a hand with exactly 2 kings}\}$$
$$Q = \{x \mid x \text{ is a hand with exactly 3 queens}\}$$

$$n(K \cup Q) = n(K) + n(Q) - n(K \cap Q) = 103,776 + 4512 - 0$$
$$= 108,288$$