Exam 2

Math 105, Summer 2018

Name: Key

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 82 points on this exam. That means you should budget about 0.6 minute(s) for each point a problem is worth in order to complete the exam in time.

Notes: You need to use the *words* "true" or "false" in the true/false section, rather than using the letters 'T' or 'F.' Also, the final problem is worth 24 points, so be sure to allocate proper time for it.

Reminder. There are to be no devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

1. (12 points) True or False: Write the word "true" or the word "false" in the blank, depending on if the statement is true or false.

   Use the universe $U = \mathbb{R}$ (i.e. U is the set of all real numbers) and the following set names

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{x \mid x \text{ is a whole number and } -2 \leq x \leq 4\}$$

   (a) True $2 \in A$

   (b) False $-5 \in B$

   (c) True $6 \notin A$

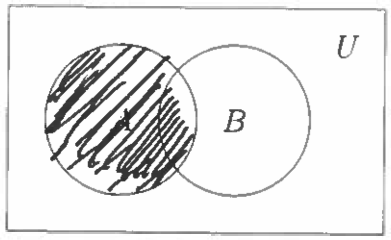
   (d) False $3 \notin B$

   (e) True $A \cup B = \{x \mid x \text{ is a whole number and } -2 \leq x \leq 5\}$

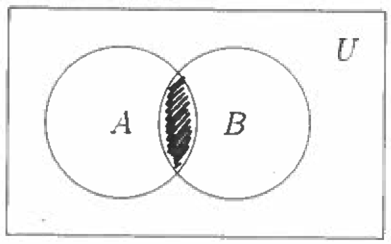
   (f) False $A \cap (B') = \{3, 5\}$

2. (8 points) Given the following Venn Diagram, shade the specified set.

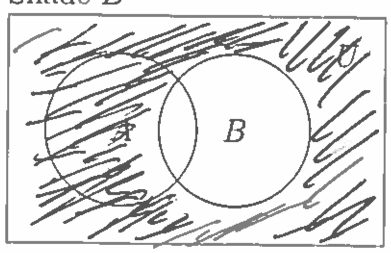
(a) Shade A



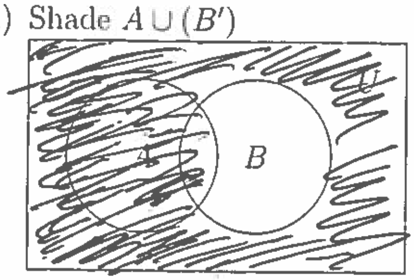
(b) Shade $A \cap B$



(c) Shade B'



(d) Shade $A \cup (B')$



3. (8 points) Consider the universe, $U = \{0, 1, 2, 3, 4, 5\}$

   (a) Give examples of sets A and B so that $A \cup B = A$ (i.e you should define a set called A and a set called B and you should make sure that $A \cup B = A$).

   $A = \{0, 1\}$

   $B = \{0\}$

   $A \cup B = \{0, 1\} = A$ (There are many more correct answers)

   (b) Give examples of sets A and B so that $n(A \cup B) \neq n(A) + n(B)$

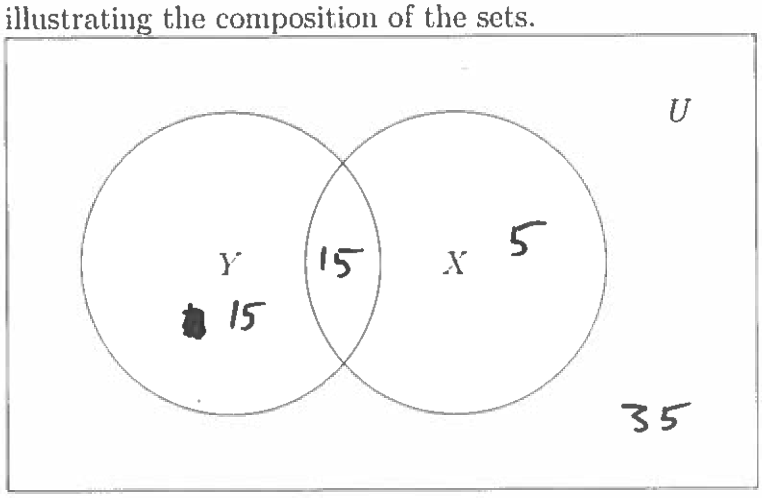
   $A = \{0, 1\}$

   $B = \{0\}$

   $A \cup B = \{0, 1\}$

   $n(A \cup B) = 2 \neq 2 + 1 = n(A) + n(B)$

4. (8 points) You have a universe, U, and sets X and Y. You know that $n(X) = 20$, $n(Y) = 30$, $n(X \cup Y) = 35$, and $n(X') = 50$. Fill out the following Venn Diagram illustrating the composition of the sets.



$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$35 = 20 + 30 - n(X \cap Y)$

$= 50 - n(X \cap Y)$

$\rightarrow n(X \cap Y) = 15$

5. (6 points) You need to climb a tower to get the crown at the top. There are three staircases at the base of the tower. Each staircase has two hallways at the top, and each hallway ends in four magical elevators. How many different paths to the top of the tower are there?

$\underset{\text{stairs}}{\underline{\ 3\ }}\quad \underset{\text{hallways}}{\underline{\ 2\ }}\quad \underset{\text{elevators}}{\underline{\ 4\ }}$ → There are $3 \cdot 2 \cdot 4 = 24$ different paths

6. (8 points) I have 10 colored pens with which to grade this exam. Each page of the exam will be graded with its own color, there are 5 pages in this exam, and each color can only be used once. How many different color schemes do I have for grading this exam? Assume that grading the first page in green, then the second page in purple is a different color scheme than grading the first in purple and the second in green.

- drawing is without replacement
- order matters

There are ${}_{10}P_5 = 30{,}240$ different color schemes

7. (8 points) You have 15 songs in your music library. On your trip to go hiking, you have time to listen to 5 of them. How many different sets of songs can you listen to on your trip? Assume that you can only listen to a given song once and you don't care in which order you listen to the songs. Listening to "Viva la Vida" then "Sparks" is the same as listening to "Sparks" then "Viva la Vida."

- drawing is without replacement
- order does not matter

There are ${}_{15}C_5 = 3003$ different sets of songs

8. (24 points) Recall that a standard deck of cards has 52 cards: 13 each of hearts, spades, clubs, and diamonds, and 4 of each of the denominations (2–10, jack, queen, king, ace). In how many ways can you be dealt a 5 card hand with...

   (a) ...1 diamond and 4 clubs?

   $\underset{\text{diamond}}{\underline{{}_{13}C_1}} \quad \underset{\text{clubs}}{\underline{{}_{13}C_4}}$ → There are 9295 different hands

   (b) ...1 diamond?

   $\underset{\text{diamond}}{\underline{{}_{13}C_1}} \quad \underset{\text{not}}{\underline{{}_{39}C_4}}$ → There are 1,069,263 different hands

   (c) ...4 clubs?

   $\underset{\text{clubs}}{\underline{{}_{13}C_4}} \quad \underset{\text{not}}{\underline{{}_{39}C_1}}$ → There are 27,885 different hands

   (d) ...1 diamond or 4 clubs?

   $D = \{x \mid x \text{ is a hand with 1 diamond}\}$
   $C = \{x \mid x \text{ is a hand with 4 clubs}\}$

   $$\begin{aligned} n(D \cup C) &= n(D) + n(C) - n(D \cap C) \\ &= 1{,}069{,}263 + 27{,}885 - 9295 \\ &= 1{,}087{,}853 \text{ different hands} \end{aligned}$$