Exam 3

Math 105. Summer 2019

Name: Key

Don't leave anything blank: If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work: If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself: If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 92 points on this exam. That means you should budget about 0.5 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder: There are to be no devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

Note: If you use any set names that I haven't defined, you need to define them. If you choose not to use set names, you need to give me a brief description of what you are counting.

1. **Vocabulary** (5 points each)

(a) Given an experiment, what is the sample space?

The sample space is the set of all possible outcomes

(b) Given an experiment, what is an event?

An event is a subset of the sample space

(c) What does it mean for two events, E and F, to be mutually exclusive?

E and F are mutually exclusive if $E \cap F = \phi$.

(d) State the Law of Large Numbers.

The Law of Large Numbers states that if you run an experiment many times, the relative frequency of any event will approach the probability of that event.

2. Consider the experiment where you roll two *four-sided* dice.

   (a) (5 points) What is the sample space of the experiment?

   $S = \{ (1,1), (1,2), (1,3), (1,4),$
   $(2,1), \dots, (2,4)$
   $\vdots$
   $(4,1), \dots, (4,4) \}$

   (b) (5 points) Find the event where you roll an even number on the first die and an odd number on the second die.

   $E = \{ (2,1), (2,3), (4,1), (4,3) \}$

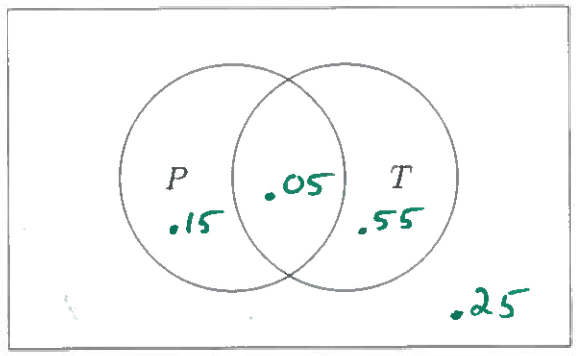
   (c) (3 points) What are the *odds* that you roll an even number on the first die and an odd number on the second die?

   $o(E) = 4 : 12$

   (d) (3 points) What are the *odds* that you roll an odd number on the first die or an even number on the second die?

   $o(E') = 12 : 4$

3. (12 points) You are stranded on a deserted island. The probability that a passing plane sees you and saves you is .2. The probability that sea turtles carry you off the island to safety is .6. The probability that neither a plane nor a sea turtle saves you is .25. Letting P be the event that a plane saves you and T be the event that sea turtles save you, fill out the following Venn Diagram illustrating the probabilities of the various regions in the Venn Diagram.



$$p(P \cup T) = .75 = p(P) + p(T) - p(P \cap T)$$
$$= .2 + .6 - p(P \cap T)$$
$$= .8 - p(P \cap T)$$

$$\Rightarrow p(P \cap T) = .05$$

4. Recall the composition of a standard deck of cards: each card has a denomination and a suit. There are 13 denominations (2–10, jack, king, queen, and ace) and 4 suits (clubs, spades, diamonds, and hearts), yielding 52 cards in total. You are dealt a 5-card hand.

   (a) (8 points) What is the probability of being dealt a hand with exactly 3 hearts and no clubs?

$H = \{x \mid x \text{ is a hand with exactly 3 hearts}\}$

$C = \{x \mid x \text{ is a hand with no clubs}\}$

$$n(H \cap C) = \underbrace{{}_{13}C_3}_{\text{hearts}} \; \underbrace{{}_{26}C_2}_{\substack{\text{not hearts} \\ \text{not clubs}}} = 92950$$

$$p(H \cap C) = \frac{92950}{2598960} \approx .036$$

   (b) (12 points) What is the probability of being dealt a hand with exactly 3 hearts or no clubs?

$$n(H) = \underbrace{{}_{13}C_3}_{\text{hearts}} \cdot \underbrace{{}_{39}C_2}_{\text{not hearts}} = 211926$$

$$n(C) = {}_{39}C_5 = 575757$$

$$n(H \cup C) = n(H) + n(C) - n(H \cap C)$$

$$= 211926 + 575757 - 92950$$

$$= \underline{694733}$$

$$p(H \cup C) = \frac{694733}{2598960} \approx .27$$

5. You are running for the executive board in Digitopolis' election. Digitopolis runs their elections so that the candidate who receives the most votes becomes Mathemagician, the candidate who receives the second-most votes becomes Leading Statistician, and the candidate who receives the third-most votes becomes the Applied Mathematician. There are 11 total candidates.

(a) (8 points) What is the probability that you are elected to one of the three positions?

# of exec boards : ${}_{11}P_3 = 990$

# of ways you are elected Mathemagician : ${}_{10}P_2 = 90$

" " L.S. : ${}_{10}P_2 = 90$

" " A.M. : ${}_{10}P_2 = 90$

# of ways you are elected : 270

probability you are elected : $\frac{270}{990} = .27$

(b) (16 points) Suppose that the Mathemagician earns \$50,000 per year, the Leading Statistician earns \$40,000 per year, the Applied Mathematician earns \$30,000 per year, and everyone else in Digitopolis earns \$20,000 per year. What is the expected value of your salary next year?

event	M	LS	AM	other
probability	90/990	90/990	90/990	720/990
value	50000	40000	30000	20000

$$E.V. = (90/990)(50000) + (90/990)(40000) + (90/990)(30000) + (720/990)(20000)$$

$$\approx 25454.54$$

6. Bonus: (possible 5 points) Consider the following experiment: you pick a random natural number (i.e. a nonnegative whole number: $0, 1, 2, 3, \dots$). Use complete sentences in all parts.

   (a) What do you think the probability of randomly choosing an even number should be? Why?

   (b) Are the methods that we currently have for computing probabilities adequate for computing the probability of drawing an even number? Why or why not?

   (c) If our methods are not adequate, can you come up with a rigorous method for computing the probability of randomly drawing an even number?