KEY

Are the following arguments valid or invalid? For problem 1, use both the tree method and a truth table.

**Ex 1**   1. $p \leftrightarrow q \equiv \left((p \wedge q) \vee ((\sim p) \wedge (\sim q)) \right)$ ✓
          2. $q \vee (\sim p)$ ✓

$\sim (q \wedge p) \equiv (\sim q) \vee (\sim p)$ ✓

```
              ~(q∧p)
          /          \
         p            ~p
         q            ~q
       /    \        /    \
      q     ~p      q     ~p
      |      X      X    /  \
    /   \               ~q   ~p
   ~q   ~p
   X    X
```
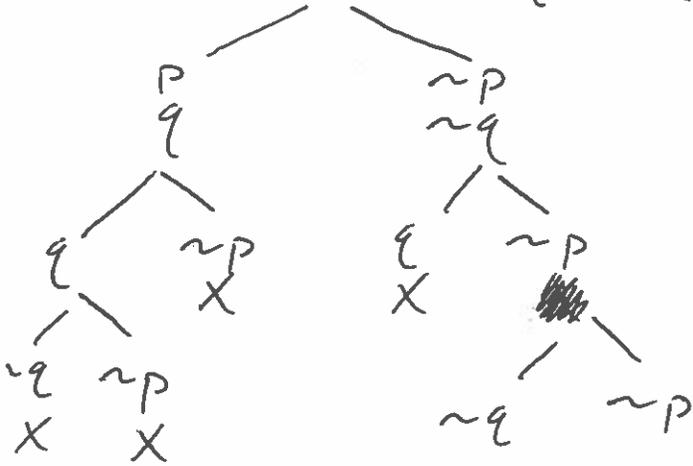
The argument is invalid

Ex 2   1. $(p \vee r) \to q$ ✓
          2. $(\sim r) \vee (\sim q)$ ✓

$\sim (\sim r) \equiv r$ ✓

```
         ~(~r) ≡ r
        /         \
   ~(p∨r)          q
     ~p          /    \
     ~r         ~r    ~q
     X          r     X
                X
```
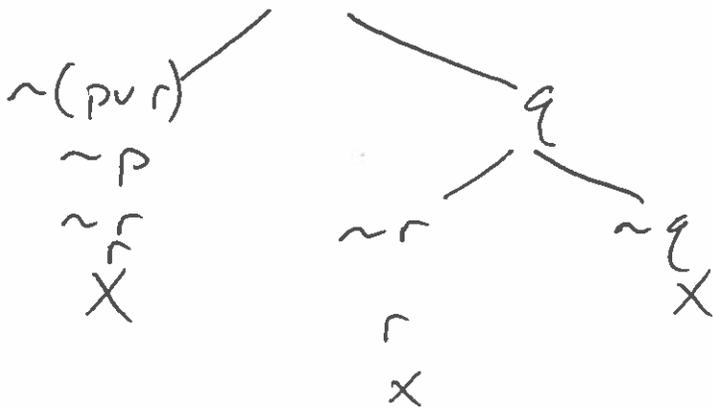
The argument is valid

1

Ex 3 1. $(p \to q) \vee (\sim r)$ ✓
2. $\sim (r \vee (q \wedge s)) \equiv (\sim r) \wedge (\sim(q \wedge s))$ ✓
3. $s \to ((\sim r) \to (\sim p))$ ✓
$\sim \therefore [(p \vee q) \to (r \vee s)]$ ✓

$p \vee q$ ✓
$\sim(r \vee s)$ ✓
$\sim r$
$\sim s$

Left branch:

p
$\sim r$
$\sim(q \wedge s)$ ✓

$\sim q$ / $\sim s$

Under $\sim q$:
$p \to q$ $\sim r$
$\sim p$ q
X   X

$\sim s$ $(\sim r) \to (\sim p)$

Under $\sim s$:
$p \to q$ $\sim r$
$\sim p$ q
X

Right branch:

$\sim q$
$\sim r$
$\sim(q \wedge s)$ ✓

$\sim q$ / $\sim s$
X

$p \to q$ $\sim r$
$\sim p$ q

↑
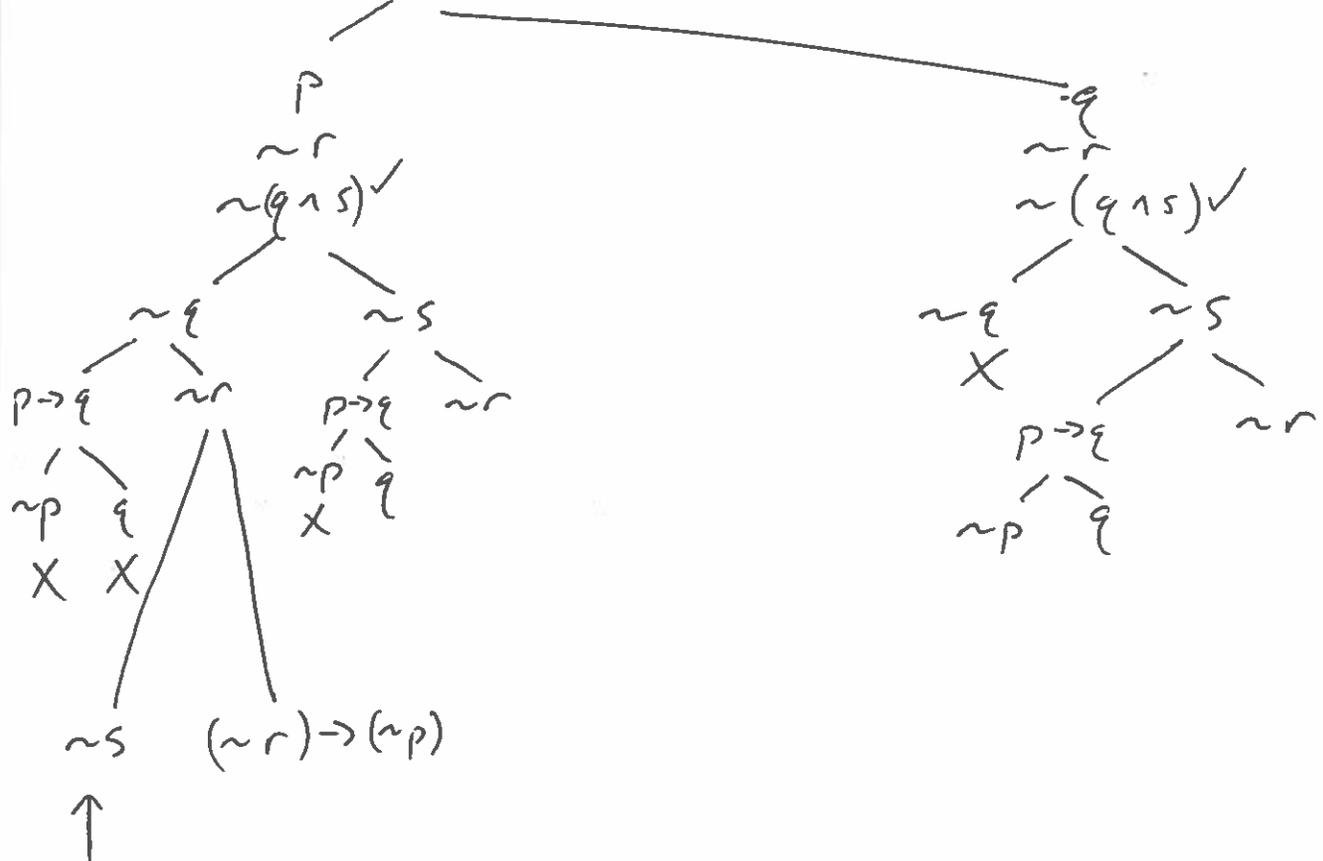
open branch,
so argument is invalid and
we don't need to complete
the rest of the tree