Final Exam Review

General Information

The exam will take place on Friday, August 16 at 10:00 am in Anstett 193 (in the usual classroom at the usual time). You will be permitted to use a calculator, but you will not be permitted to use any devices with internet access. You must bring your own calculator if you wish to use one. You will be permitted to use a 3×5 notecard with whatever notes you want on it. In addition to using this review guide, you should study the example problems we've done in class, quick hit problems, homework problems, and lecture guide problems. You should be prepared to answer questions about...

Vocabulary

- Chapter 1
  - Section 1: syllogism, premise, conclusion, deductive, inductive, (in)valid
  - Section 2: statement, simple statement, logical connective, compound statement, negation, conjunction, disjunction, conditional, antecedent, consequent
  - Section 3: truth value, logical equivalence, De Morgan's Laws
  - Section 4: inverse, converse, contrapositive, biconditional
  - Section 5: -
- Chapter 2
  - Section 1: set, element, member, empty set, cardinality, (proper) subset, intersection, mutually exclusive, union, complement
  - Section 2: De Morgan's laws
  - Section 3: Fundamental Principle of Counting, factorial
  - Section 4: permutations, combinations, distinguishable permutations
- Chapter 3
  - Section 2: experiment, outcome, sample space, event, probability, impossible event, certain event, odds, relative frequency, law of large numbers
  - Section 3: mutually exclusive
  - Section 4: -
  - Section 5: expected value
  - Section 6: conditional probability
  - Section 7: (in)dependent

Topics

- Formal Logic
  - Inductive and deductive arguments
  - Valid and invalid arguments
    - Venn Diagrams
    - Truth tables

- Logical connectives
- Translation (both directions)
- Logical equivalence
- Variations of the conditional

- Sets
  - Notation
  - Membership
  - Cardinality
  - Set operations/relations
    - Subsets
    - Intersections
    - Unions
    - Complements
  - Union/Intersection Rule for counting
  - Complement Rule for counting
  - Venn Diagrams
  - De Morgan's Laws

- Combinatorics
  - The Fundamental Principle of Counting
  - Counting techniques
    - With or without replacement?
    - Does order matter?
    - Permutations
    - Combinations
  - Distinguishable Permutations

- Probability
  - Sample spaces
  - Events
  - Probabilities
  - Odds
  - Law of Large Numbers
  - Union/Intersection Rule for probability
  - Complement Rule for probability
  - Product Rule for probability
  - Mutually exclusive events
  - Conditional probability
  - Independent events
  - Expected Value

Final Exam Review

Practice Problems

Logic

1. Write the following statements in symbolic form. Be sure to define clearly each letter you use.

   (a) "Cold weather is necessary for snow."

   c: it is cold
   s: it is snowing

   $s \to c$

   (b) "Sunny weather is sufficient for it to not rain"

   s: it is sunny
   r: it is raining

   $s \to r$

2. Consider the statements:

   p: It is humid.
   q: It is warm.
   r: It is summer.
   s: The sun is shining.

   (a) Translate the following symbolic statements into natural language sentences:

   i. $(r \wedge p) \wedge (q \vee (\sim s))$

   It [redacted] is summer, it is humid, and it is either warm, or the sun is not shining

   ii. $r \to (q \wedge (\sim p))$

   If it is summer, then it is warm and not humid.

   (b) Translate the following natural language statements into symbolic statements:

   i. "It's either not warm or it's humid if it's not summer."

   $(\sim r) \to ((\sim q) \vee p)$

   ii. "It is not the case that it is summer, the sun is shining, and it is cool."

   $\sim (r \wedge (s \wedge (\sim q)))$

   or

   $(\sim r) \wedge (s \wedge (\sim q))$ are fine

   Th. ~~statement~~ is ambiguous

3. Construct a truth table for the symbolic statement $(p \wedge q) \wedge ((\sim p) \vee (\sim q))$. Is the statement a tautology, a contradiction, or neither?

p	q	$p \wedge q$	$\sim p$	$\sim q$	$((\sim p) \vee (\sim q))$	$(p \wedge q) \wedge ((\sim p) \wedge (\sim q))$
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

contradiction

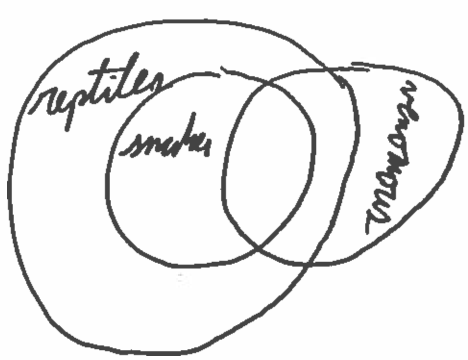
4. Use De Morgan's Laws to write a sentence that represents the *negation* of the following statement: "I either practice or I make mistakes."

I don't practice and I don't make mistakes

5. Construct a Venn diagram to determine the validity of the following argument.

1. Some snakes are venomous.
2. All snakes are reptiles.

$\therefore$ Some reptiles are venomous.



The argument is valid

6. For the following argument: define the necessary symbols, rewrite the argument in symbolic form, and then use a truth table or the tree method to determine whether or not the argument is valid.

   1. Having syrup or jam is necessary for eating pancakes.
   2. I don't have syrup.

   ∴ I am not eating pancakes.

s: I have syrup
j: I have jam
p: I am eating pancakes

1. $p \to (s \vee j)$

2. $\sim s$

$\therefore \sim p$

Statement: $(① \wedge ②) \to \therefore$

p	s	j	$s \vee j$	① $p \to (s \vee j)$	② $\sim s$	$\therefore$ $\sim p$	$① \wedge ②$	$(① \wedge ②) \to \therefore$
T	T	T	T	T	F	F	F	T
T	T	F	T	T	F	F	F	T
T	F	T	T	T	T	F	T	F
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	T	F	T
F	T	F	T	T	F	T	F	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

The argument is invalid

Sets

1. Consider the universe, $U = \{1, 2, 3, 4, 5, 6, 7\}$. Give an example of sets A and B with the listed properties, or state that such sets do not exist and explain why.

   (a) $A \cap B = \varnothing$

   $A = \{1, 2\}$

   $B = \{3\}$

   (b) $A \subseteq B$

   $A = \{1\}$

   $B = \{1, 2\}$

   (c) $1 \in A \cap B$, $2 \in A$, and $2 \notin A \cap B$

   $A = \{1, 2\}$

   $B = \{1\}$

   (d) $A \cap B = \{2, 3, 5\}$ and $A \cup B = \{2, 3, 4\}$

   Such sets are impossible to construct, because $A \cap B$ is always a subset of $A \cup B$, but this is not the case in the hypotheses.

   (e) $n(A) = 3$, $n(B) = 2$, $n(A \cup B) = 6$

   Such sets are impossible to construct because if A has 3 elements and B has 2 elements, then $A \cup B$ has at most $3 + 2 = 5$ elements.

   (f) $n(A \cup B) = 3$, $n(A) = 3$, and $n(B) = 2$.

   $A = \{1, 2, 3\}$

   $B = \{1, 2\}$

2. Given a universe, U, along with sets A and B, suppose that $n(U) = 75$, $n(A) = 33$, $n(B) = 23$, and $n(A \cup B) = 45$. Find $n(A \cap B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$45 = 33 + 23 - n(A \cap B)$$
$$= 58 - n(A \cap B)$$

Hence, $n(A \cap B) = 13$

3. For the following problems, use the sets $U = \{O, L, I, V, E, S\}$, $A = \{L, O, V, E\}$, and $B = \{L, I, V, E\}$.

   (a) Find $A \cap B$.

   $$A \cap B = \{L, V, E\}$$

   (b) Find $n(A' \cup B')$. $= n((A \cap B)') = n(U) - n(A \cap B)$
   $$= 6 - 3$$
   $$= 3$$

4. In a survey about crepe preferences, 70 people like sweet crepes, 50 like savory crepes, and 10 people liked neither. If 100 people were surveyed, how many people liked both sweet and savory crepes?

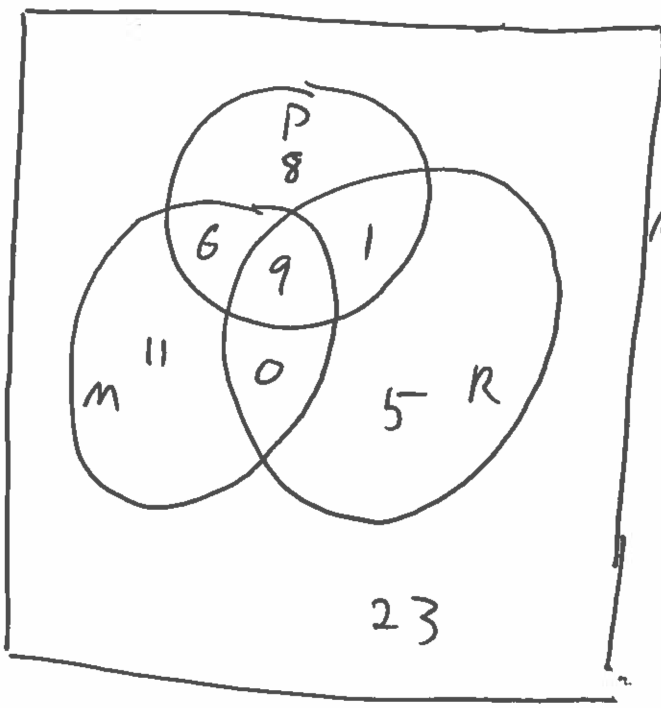
$W = \{x \mid x \text{ likes sweet crepes}\}$
$A = \{x \mid x \text{ likes savory crepes}\}$

$$n(W \cup A) = n(W) + n(A) - n(W \cap A)$$
$$90 = 70 + 50 - n(W \cap A)$$
$$= 120 - n(W \cap A)$$

Hence, 30 people like both sweet and savory crepes.

5. At a language conference, 24 of the attendees are fluent in Portuguese, 26 are fluent in Mandarin, and 15 are fluent in Russian. It turns out that 9 of the attendees are fluent in all 3 languages, 23 are fluent in none of the three (Portuguese, Mandarin, or Russian), 10 people are fluent in both Portuguese and Russian, 6 people are fluent in both Portuguese and Mandarin but not Russian, and 5 are fluent in Russian but not Portuguese or Mandarin.

   (a) Draw a Venn diagram illustrating the languages spoken by the conference attendees.



$P = \{x \mid x \text{ speaks Portuguese}\}$

$R = \{x \mid x \text{ speaks Russian}\}$

$M = \{x \mid x \text{ speaks Mandarin}\}$

   (b) What percentage of the attendees were fluent in all three (Portuguese, Mandarin, and Russian)?

$$\frac{9}{9+1+6+8+11+5+23} = \frac{9}{63} \approx .14$$

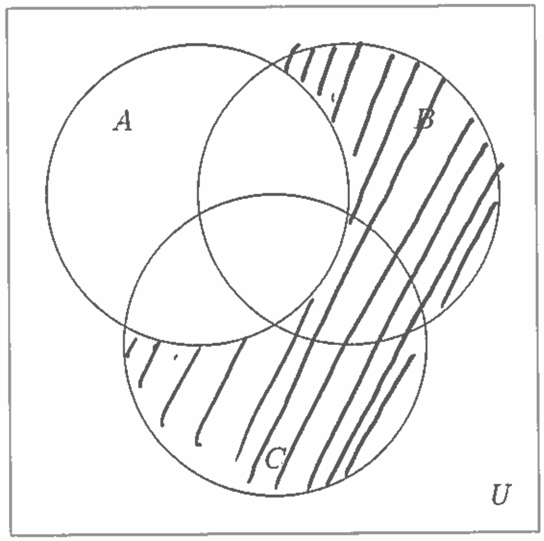
About 14% of attendees spoke all 3 languages

   (c) What percentage of the attendees were fluent in Mandarin but not Portuguese or Russian?

$$\frac{11}{63} \approx .17$$

About 17% of attendees spoke Mandarin, but not Portuguese or Russian

6. In the Venn diagram below, shade the set $A' \cap (B \cup C)$:



Combinatorics

1. Find the number of distinct arrangements of the letters in the word BANANAS.

7 letters
3 As
2 Ns

$\longrightarrow \dfrac{7!}{3!\,2!} = 420$ ~~all~~ different arrangements

2. Find the number of distinct arrangements of the letters in the word ARRANGEMENT.

11 letters
2 As
2 Rs
2 Ns
2 Es

$\longrightarrow \dfrac{11!}{2!\,2!\,2!\,2!} = 2,494,800$ different arrangements

3. A secure online banking site requires a password consisting of 6 digits or letters followed by one of the four symbols *, $, !, or %. How many possible passwords of this form are there?

$\underbrace{\underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}}_{\text{digits/letters}}\ \underbrace{\underline{4}}_{\text{symbol}} = 8,707,129,344$ different passwords

4. A group of 6 men and 7 women must form a 4-person committee. How many committees consist of 2 or more men?

committees with 2 men: ${}_6C_2 \cdot {}_7C_2 = 315$

committees with 3 men: ${}_6C_3 \cdot {}_7C_1 = 140$

committees with 4 men: ${}_6C_4 \cdot {}_7C_0 = 15$

committees with 2 or more men: $315 + 140 + 15 = 470$

5. You are deciding what you want to eat tomorrow. For breakfast, you can have a bagel, cereal, or a muffin. For lunch, you can have a sandwich or a salad. For dinner, you can have tacos, meatloaf, spaghetti, chicken nuggets, or pulled pork.

(a) How many different meal combinations can you have?

$\underset{B}{\underline{3}}\ \underset{L}{\underline{2}}\ \underset{D}{\underline{5}} = 30$ different options

(b) How many different meal combinations can you have where you eat a bagel for breakfast and a salad for lunch?

$\underset{B}{\underline{1}}\ \underset{L}{\underline{1}}\ \underset{D}{\underline{5}} = 5$ different options

(c) How many different meal combinations can you have where you eat either a bagel or a muffin for breakfast?

$\underset{B}{\underline{2}}\ \underset{L}{\underline{2}}\ \underset{D}{\underline{5}} = 20$ different options

(d) How many different meal combinations can you have where you eat either a bagel for breakfast or chicken nuggets for dinner?

bagel: $\underline{1}\ \underline{2}\ \underline{5} = 10$

nuggets: $\underline{3}\ \underline{2}\ \underline{1} = 6$

both: $\underline{1}\ \underline{2}\ \underline{1} = 2$

Number of combinations with either a bagel for breakfast or nuggets for dinner: $10 + 6 - 2 = 14$

6. You decide that you want to complete the final exam for this course one page at a time, in a random order. Suppose that the final exam has 8 pages.

   (a) In how many different ways can you complete the exam?

   $\underset{\text{first}}{\underline{8}}\ \underset{\text{second}}{\underline{7}}\ \underset{\text{third}}{\underline{6}}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

   $\rightarrow 8!$ ways of completing the exam

   or: ${}_8P_8 = 8!$ ways

   (b) In how many different ways can you complete 5 pages of the exam?

   $\underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4} = 6720$ ways

   or

   ${}_8P_5 = 6720$ ways

   (c) In how many different ways can you complete the exam if you want to do the first page last?

   $\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1} = 7!$ ways

   (d) In how many different ways can you complete the exam if you want to do the first page last or the second page third?

   1^{st} last and 2^{nd} third: $\underline{6}\ \underline{5}\ \underline{1}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1} = 6!$

   1^{st} last: $7!$ (see (c))

   2^{nd} third: $\underline{7}\ \underline{6}\ \underline{1}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 7!$

   1^{st} last or 2^{nd} third: $7! + 7! - 6! = 9360$ ways

7. You are dealt a 5-card hand from a standard deck of cards (count an ace as high).

   (a) How many hands have exactly 4 cards below a 7 and exactly 1 card above a 10?

$$\underset{<7}{\underline{{}_{20}C_4}} \quad \underset{>10}{\underline{{}_{16}C_1}} = 77,520 \text{ hands}$$

   (b) How many hands have exactly two hearts or no queens?

$H = \{x \mid x \text{ is a hand with exactly 2 hearts}\}$

$Q = \{x \mid x \text{ is a hand with no queens}\}$

$n(H) = \underline{{}_{13}C_2} \; \underline{{}_{39}C_3} = 712842$

$n(Q) = {}_{48}C_5 = 1712304$

$$n(H \cap Q) = \underset{\text{hearts, not queen}}{\underline{{}_{12}C_2}} \quad \underset{\text{other, not queen}}{\underline{{}_{36}C_3}} = 471240$$

$n(H \cup Q) = 712842 + 1712304 - 471240 = 1953906$

   (c) How many hands have exactly two hearts and exactly two queens?

Case 1: 2 hearts, 2 queens, and queen of hearts

$$\underset{Q\heartsuit}{\underline{1}} \quad \underset{\text{hearts, not queen}}{\underline{{}_{12}C_1}} \quad \underset{\text{queens, not hearts}}{\underline{{}_{3}C_1}} \quad \underset{\text{not hearts, not queens}}{\underline{{}_{36}C_2}} = 22680$$

Case 2: 2 hearts, 2 queens, no queen of hearts

$$\underset{\text{hearts, not queen}}{\underline{{}_{12}C_2}} \quad \underset{\text{queens, not hearts}}{\underline{{}_{3}C_2}} \quad \underset{\text{not hearts, not queens}}{\underline{{}_{36}C_1}} = 7128$$

Total: $22680 + 7128 = \boxed{29808 \text{ hands}}$

Probability

1. A single six-sided die is rolled. Find the probability of rolling a number that is both even and odd.

This can't happen, so the probability is 0.

2. A pair of six-sided dice are rolled. Find the probability that the sum is equal to 5 or 9.

$E = \{(1,4), (2,3), (3,2), (4,1), (3,6), (4,5), (5,4), (6,3)\}$

$p(E) = \frac{8}{36}$

3. On a marketing exam, a student randomly guesses the answers to 6 multiple choice problems, each with 5 possible answers (a) through (e). What is the probability that the student gets at least one of the answers incorrect?

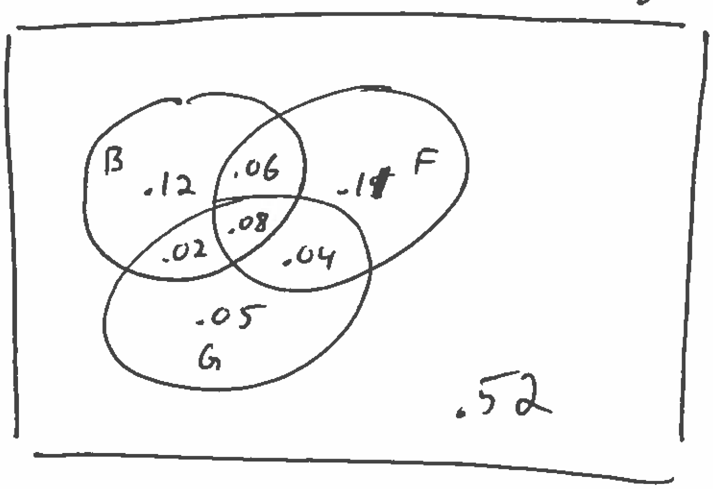
# of ways of guessing: 5^6
# of ways of getting all correct: 1
# of ways of getting at least one wrong: $5^6 - 1$
probability of getting at least one wrong: $\frac{5^6-1}{5^6} \approx .9999$

4. A survey of a group of students' viewing habits over the last year revealed the following information:

- 28% of those surveyed watched basketball
- 29% of those surveyed watched football
- 19% of those surveyed watched gymnastics
- 14% of those surveyed watched basketball and football
- 12% of those surveyed watched football and gymnastics
- 10% of those surveyed watched basketball and gymnastics
- 8% of those surveyed watched all three sports.

Find the probability that a person surveyed watched none of these three sports.

$B = \{x \mid x \text{ watched basketball}\}$ $F = \{x \mid x \text{ watched football}\}$ $G = \{x \mid x \text{ watched golf}\}$



$$p(B' \cap G' \cap F') = 1 - .12 - .06 - .02 - .08 - .11 - .04 - .05$$

$$= .52$$

5. Two six-sided dice are rolled. What is the probability of rolling two distinct numbers?

$D = \{(x,y) \mid x = y\}$

$n(D) = 6$

$n(D') = 36 - 6 = 30$

$p(D') = \frac{30}{36}$

6. A friend asks you to play a game in which a single d6 is rolled. If the number rolled is odd, you win twice the number in dollars (e.g. if the roll is a 3 you win \$6). If the number rolled is even, you lose \$5. Calculate the expected value of playing the game. Is the game worth playing?

outcome	1	3	5	even
prob.	1/6	1/6	1/6	3/6
value	2	6	10	-5

$EV = (1/6)(2) + (1/6)(6) + (1/6)(10) + (3/6)(-5)$

$= .5$

The game is worth playing

7. Suppose that for some event E, $o(E) = 7 : 1000$. Find $p(E')$.

$n(E) = 7, \quad n(E') = 1000, \quad n(S) = 1007$

$p(E') = \frac{1000}{1007}$

8. Calculate the expected value of buying a \$2 lottery ticket for a 5/36 lottery if the first prize (i.e. guessing all 5 winning numbers) is worth \$500,000 and the second prize (i.e. guessing 4 of the 5 winning numbers) is worth \$1,000. Based on this calculation, is it worth buying the ticket?

F – winning first prize

T - winning second prize

$p(F) = \frac{1}{{}_{36}C_5} \approx .0000027$

$p(T) = \frac{{}_5C_4 \cdot {}_{31}C_1}{{}_{36}C_5} \approx .00041$

$p(F' \cap T') = 1 - p(T) - p(F) \approx .9996$

$EV = (.0000027)(500000) + (.00041)(1000) + (.9996)(0) - 2$

$= -.24$

It is not worth buying a ticket

9. A University of Oregon commuter survey asked students and faculty/staff about their primary mode of transportation to and from campus. The results are recorded in the following table:

	Live on campus	Walk	Bike	Bus	Car	Other	
Students	205	169	167	129	127	11	808
Faculty/Staff	0	29	94	36	231	12	402
							1210

(a) What is the probability that a person surveyed primarily rides their bike?

$B = \{x \mid x \text{ bikes to campus}\}$

$$p(B) = \frac{n(B)}{n(S)} = \frac{167+94}{1210} = \frac{261}{1210} \approx .22$$

(b) What is the probability that a person is a student, given that they live on campus?

$T = \{x \mid x \text{ is a student}\}$

$C = \{x \mid x \text{ lives on campus}\}$

$$p(T \mid C) = \frac{n(T \cap C)}{n(C)} = \frac{205}{205} = 1$$

(c) Are the events "a person is a student" and "a person walks to campus" independent or dependent? $W = \{x \mid x \text{ walks to campus}\}$

$$P(T \mid W) = \frac{n(T \cap W)}{n(W)} = \frac{169}{198} \approx .85$$

$$p(T) = \frac{n(T)}{n(S)} = \frac{808}{1210} \approx .67$$

T and W are dependent

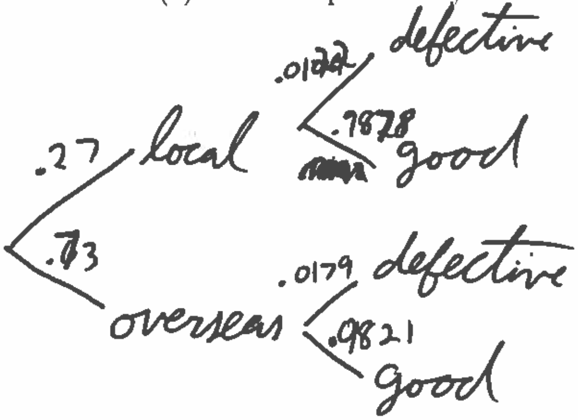
(d) What is the probability that a person bikes or rides the bus to campus given that they are a student?

$R = \{x \mid x \text{ rides the bus to campus}\}$

$$p(B \cup R \mid T) = \frac{n((B \cup R) \cap T)}{n(T)} = \frac{167+129}{808} \approx .37$$

10. A calculator manufacturer gets components from two suppliers, 27% of the components come from a local supplier and the rest come from overseas. Of the components from overseas, 1.79% are defective, while only 1.22% of the components from the local supplier are defective.

(a) Find the probability that a component is from the local supplier and is defective.



$L = \{x \mid x \text{ is from the local supplier}\}$

$D = \{x \mid x \text{ is defective}\}$

$$p(L \cap D) = (.27)(.0122)$$
$$= .003294$$

(b) Find the probability that a randomly chosen component is defective.

$$p(D) = (.27)(.0122) + (.73)(.0179)$$
$$= .016361$$

(c) Are the events "a component is from overseas" and "a component is defective" independent? Justify your answer with the appropriate calculations.

$$p(L') = .73$$

$$p(L' \mid D) = \frac{p(L' \cap D)}{p(D)} = \frac{(.73)(.0179)}{.016361} \approx .799$$

L' and D are dependent events