Lecture Examples

Ex 1 You roll a pair of six-sided die.

(a) What is the probability that their sum is 13?

$p(\emptyset) = 0$

(b) What is the probability that their sum is between 0 and 50?

$p(S) = 1$

(c) What is the probability that their sum is 8?

$E = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{5}{36} \approx .14$

Ex 2 You roll a pair of six-sided die.

(a) What is the probability their sum is even?

$E = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$p(E) = \frac{n(E)}{n(S)} = \frac{18}{36} = .5$

(b) What is the probability their sum is smaller than 4?

$F = \{(1,1), (1,2), (2,1)\}$

$p(F) = \frac{n(F)}{n(S)} = \frac{3}{36} \approx .08$

(c) What is the probability their sum is even or smaller than 4?

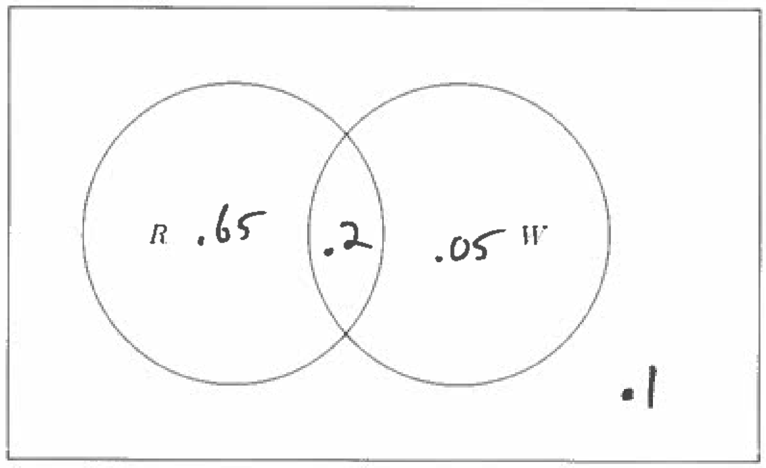
$p(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{36} \approx .03$

(d) What is the probability their sum is smaller than 11?

$G = \{(5,6), (6,5), (6,6)\}$

$p(G') = \frac{n(G')}{n(S)} = \frac{n(S) - n(G)}{n(S)} = \frac{36-3}{36} = \frac{33}{36} \approx .92$

Ex 3 The weather report says there's an 85% chance of rain, 25% chance of strong wind, and 10% chance of clear skies with no strong wind. Fill out a Venn Diagram illustrating the probabilities of each possible outcome. What is the probability of rain and strong wind?



$$p(R \cup W) = .9 = p(R) + p(W) - p(R \cap W)$$
$$= .85 + .25 - p(R \cap W)$$
$$= 1.1 - p(R \cap W)$$

$$\Rightarrow p(R \cap W) = .2$$

On-Your-Own Examples

Ex 1 A card is dealt from a standard deck of cards. Count an ace as high. Using probability rules where appropriate, find the probability that the card is:

(a) an ace and red

$R = \{x \mid x \text{ is red}\}$ $\quad A = \{x \mid x \text{ is an ace}\}$

$$p(R \cap A) = \frac{n(R \cap A)}{n(S)} = \frac{2}{52} \approx .04$$

(b) an ace or red

$$p(R \cup A) = p(R) + p(A) - p(R \cap A) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} \approx .54$$

(c) under a 6

$X = \{x \mid x \text{ is under a } 6\}$

$$p(X) = \frac{n(X)}{n(S)} = \frac{16}{52} \approx .31$$

(d) over a 6

$$p(X') = 1 - p(X) = \frac{36}{52} \approx .31$$

(e) above a 6 and below a king

$K = \{x \mid x \text{ is below a king}\}$

$$p(K \cap X) = \frac{28}{52} \approx .54$$

(f) above a 6 or below a king

– every card is above a 6 or below a king

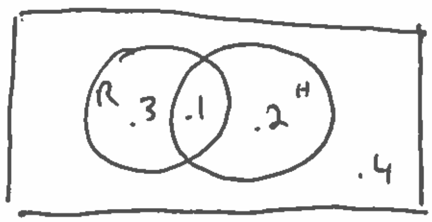
$$p(K \cup X) = 1$$

Ex 2 The weather report says there's a 40% chance of rain, a 30% chance of hail, and a 60% chance of rain or hail. Find the probability of:

$$p(R \cup H) = p(R) + p(H) - p(R \cap H)$$

$$.6 = .4 + .3 - p(R \cap H)$$

$$= .7 - p(R \cap H) \rightarrow p(R \cap H) = .1$$



(a) both rain and hail.

.1

(b) neither rain nor hail.

.4

(c) no rain.

$.2 + .4 = .6$

(d) rain but no hail.

.3

Ex 3 If $p(E) = \frac{3}{8}$, find $o(E)$ and $o(E')$.

$p(E) = \frac{3}{8} = \frac{n(E)}{n(S)}$

- This does not imply that $n(E) = 3$,
  but it does imply $o(E) = 3:5$ and $o(E') = 5:3$

Ex 4 Two six-sided dice are rolled. Find the probability that the sum of the dice is:

(a) 5

$E = \{(1,4), (2,3), (3,2), (4,1)\}$

$p(E) = \frac{4}{36} \approx .11$

(b) 11

$F = \{(5,6), (6,5)\}$

$p(F) = \frac{2}{36} \approx .06$

(c) even

See Lecture Ex 2a: b

$G = \{(a,b) \mid a+b \text{ is even}\}$

$p(G) = .5$

(d) even and doubles

$D = \{(a,b) \mid a = b\}$

$D \subseteq G \Rightarrow D \cap G = D$

$p(G \cap D) = \frac{n(G \cap D)}{n(S)} = \frac{n(D)}{n(S)} = \frac{6}{36} \approx .17$

(e) even or doubles

$D \subseteq G \Rightarrow D \cup G = G$

$p(G \cup D) = p(G) = .5$

Ex 5 Two hundred people apply to work at a company with three open identical positions. Sixty of the applicants are women. If three people are selected at random to fill the open positions, find the probability that...

(a) ...two of the three are women.

- drawing without replacement
- order doesn't matter

\# of ways of filling the positions : ${}_{200}C_3 = 1{,}313{,}400$

\# of ways where two of three are women : $\underbrace{{}_{60}C_2}_{w} \cdot \underbrace{{}_{140}C_1}_{m} = 247{,}800$

probability that two of three are women : $\frac{247{,}800}{1{,}313{,}400} \approx .19$

(b) ...none of the three are women.

\# of ways that none are women : ${}_{140}C_3 = 447{,}580$

probability that none are women : $\frac{447580}{1313400} \approx .34$

(c) ...at least one of the three is a woman.

- this is the complement of the event in (b)

probability that at least one is a women : $1 - .34 = .66$