Lecture Examples

Ex 1 You roll a six-sided die twice.

(a) What is the sample space?

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), \ldots, (3,6), (4,1), \ldots, (4,6), (5,1), \ldots, (5,6), (6,1), \ldots, (6,6)\}$$

(b) What is the set corresponding to the event "the sum of the die is a 9?"

$$E = \{(3,6), (4,5), (5,4), (6,3)\}$$

(c) What is the probability of rolling a sum of 9?

$$p(E) = \frac{4}{36} \approx .11$$

(d) What are the odds of rolling a sum of 9?

$$o(E) = n(E) : n(E') = 4 : 32$$

(e) What is the probability of rolling a sum of 12?

$$F = \{(6,6)\}$$
$$p(F) = \frac{1}{36} \approx .028$$

(f) What is the probability of rolling a sum of at least 10?

$$G = \{(4,6), (5,5), (6,4), (5,6), (6,5), (6,6)\}$$
$$p(G) = \frac{6}{36} \approx .17$$

On-Your-Own Examples

Ex 1 A sock drawer is filled with assorted striped, patterned, and solid color socks. There are 6 pairs of striped socks, 8 pairs of patterned socks, and 7 pairs of solid color socks. The socks are not matched up and a single sock is drawn at random from the drawer.

(a) What is the sample space?

$$S = \{x \mid x \text{ is a sock in the drawer}\}$$

(b) What is the probability that a striped sock is drawn?

$$T = \{x \mid x \text{ is a striped sock}\}$$
$$p(T) = \frac{12}{42} \approx .29$$

(c) What are the odds of a striped sock being drawn?

$$o(T) = n(T):n(T') = 12:30$$

(d) What is the probability that the sock drawn is not solid?

$$O = \{x \mid x \text{ is a solid sock}\}$$
$$p(O') = \frac{28}{42} \approx .67$$

Ex 2 A six-sided die is tossed twice.

(a) What is the sample space?

See lecture Ex 1 a

(b) Find the event "an even number is tossed and then an odd number."

$$E = \{(2,1), (2,3), (2,5), (4,1), (4,3), (4,5), (6,1), (6,3), (6,5)\}$$

(c) Find the probability of rolling a six twice.

$$T = \{(6,6)\}$$
$$p(T) = \frac{1}{36} \approx .028$$

(d) How likely is it that the first number is even?

$$F = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), \ldots, (4,6), (6,1), \ldots, (6,6)\}$$
$$p(F) = \frac{18}{36} = .5$$

(e) What are the odds that the first number is a six?

$$G = \{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$
$$p(G) = \frac{6}{36} \approx .17$$

(f) Find the event that the sum is 7.

$$H = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

(g) Find the probability that the sum is 7.

$$p(H) = \frac{6}{36} \approx .17$$

(h) Find the odds that the sum is 7.

$$o(H) = \cancel{} \; 6:30$$

Ex 3 A computer randomly arranges the letters in the word SUMMER. How likely is it that the random arrangement spells REMMUS? # of distinguishable permutations of "summer": $\frac{6!}{2!} =$

probability of spelling "remmus": $\frac{1}{360} \approx .0028$ 360

Ex 4 If the odds of an event E are $2 : 7$, i.e. $o(E) = 2 : 7$, what is $p(E)$?

$n(E) = 2, \; n(E') = 7$

$$p(E) = \frac{n(E)}{n(S)} = \frac{n(E)}{n(E)+n(E')} = \frac{2}{9} \approx .22$$

Ex 5 A family is planning to have four children. (Assume that boys and girls are equally likely).

(a) What is the probability that all four children are boys?

$n(S) = 2^4 = 16$

$$p(\{(B,B,B,B)\}) = \frac{1}{16} \approx .0625$$

(b) What is the probability that at least three of the children are girls?

$E = \{(B,G,G,G), (G,B,G,G), (G,G,B,G), (G,G,G,B), (G,G,G,G)\}$

$$p(E) = \frac{5}{16} \approx .3125$$

(c) What is the probability that at least one child is a girl?

probability that none are girls: $\frac{1}{16}$

probability of at least one girl: $1 - \frac{1}{16} = \frac{15}{16} \approx .9375$

(d) What are the odds that at least one child is a girl?

$15 : 1$

(e) What is the likelihood that the family has two boys and then two girls?

$$p(\{(B,B,G,G)\}) = \frac{1}{16} = .0625$$

(f) What is the likelihood that the family has two boys and two girls?

$F = \{(B,B,G,G), (B,G,B,G), (B,G,G,B), (G,B,G,B), (G,G,B,B), (G,B,B,G)\}$

$$p(F) = \frac{6}{16} \approx .375 \quad 3$$