

Practice Exam 3

Math 105, Summer 2019

Here is a list of the vocabulary you may be asked to define, along with the names of some important topics from each section

- Section 2:
 - Vocab words: experiment, outcome, sample space, event, probability, impossible event, certain event, odds, relative frequency, law of large numbers
 - Concepts: compute probability and odds of an event, understand the law of large numbers
- Section 3:
 - Vocab words: mutually exclusive
 - Concepts: know probabilities of impossible and certain events, use rules of probability and understand how they correspond to similar rules regarding sets, use Venn Diagrams to work with probability
- Section 4:
 - Vocab words: -
 - Concepts: apply counting methods to compute probabilities
- Section 5:
 - Vocab words: expected value
 - Concepts: compute and interpret expected value

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Name: _____

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 112 points on this exam. That means you should budget about 0.4 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder. There are to be no devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

Notes: Give all probabilities as decimals with two digits. Make sure to define any new letters that you use.

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1. You flip a coin, then roll a six-sided die.

(a) (6 pts) What is the sample space?

$$S = \{ (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \}$$

(b) (6 pts) Find the event that you flip heads, then roll an even number.

$$E = \{ (H, 2), (H, 4), (H, 6) \}$$

(c) (4 pts) What is the probability that you flip heads, then roll an even number?

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12}$$

(d) (4 pts) What is the probability that you either flip tails, or roll an odd number?

$$P(E') = 1 - P(E) = 1 - \frac{3}{12} = \frac{9}{12}$$

(e) (4 pts) What are the odds that you flip tails and roll at least a 5?

$$F = \{ (T, 5), (T, 6) \}$$

$$O(F) = n(F) : n(F') = 2 : 10$$

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2. Consider the sample space $S = \{1, 2, 3, 4, 5, 6\}$

(a) (4 pts) Give an example of events E and F so that $p(E \cup F) \neq p(E) + p(F)$

$$E = \{1\}$$

$$F = \{1\}$$

$$E \cup F = \{1\}$$

$$p(E \cup F) = \frac{1}{6} \neq p(E) + p(F) = \frac{1}{6} + \frac{1}{6}$$

(b) (6 pts) Are there any events, G , with $p(G) > 1$? If so, give an example. If not, explain why (using complete sentences).

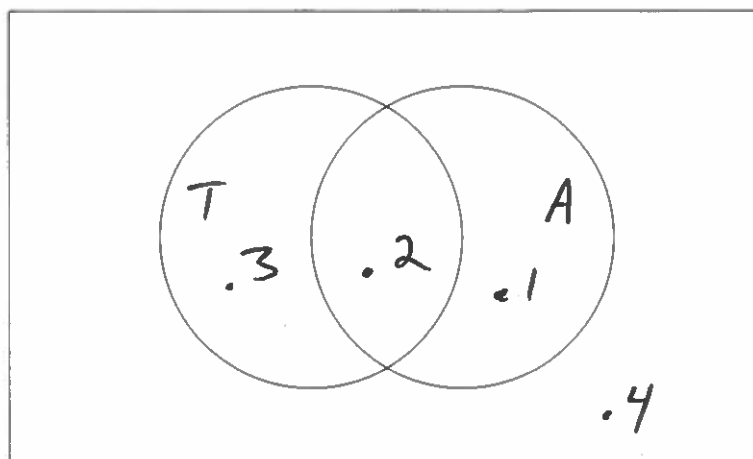
No such events exist because $0 \leq p(G) \leq 1$ for all events, G .

(c) (6 pts) Suppose the experiment is that you roll a six-sided die once. You run 10 trials of this experiment come up with seven 2s, a 1, a 3, and a 4. Does the Law of Large Numbers indicate that you are less likely to roll a 2 on your next roll because you've already rolled lots of 2s? Why or why not? Make sure to use complete sentences.

The Law of Large Numbers does not indicate that you are less likely to roll a 2 on your next ~~experiment~~ trial. The Law of Large Numbers only states that the relative frequency of rolling a 2 will eventually approach $\frac{1}{6}$.

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3. (10 pts) You are riding a train from Portland to Eugene. You are seated next to a random passenger on the train. Suppose there is a 50% chance that the person you sit next to wants to talk to you for the entire train ride, there is a 30% chance that the person believes in astrology, and there is a 10% chance that the person believes in astrology, but doesn't want to talk to you for the entire train ride. Fill out the following Venn Diagram, being sure to define your events (~~you don't need to use set notation here~~) and illustrating the probabilities of each region of the Venn Diagram.



$T = \{x \mid x \text{ wants to talk to you for the ride}\}$

$A = \{x \mid x \text{ believes in astrology}\}$

$$p(T) = .5 \quad p(A) = .3 \quad p(A \cap T') = .1$$

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4. You roll two six-sided dice. Find the probability that...

(a) (6 pts) ...you roll two even numbers.

$$E = \{ (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6) \}$$

$$P(E) = \frac{9}{36}$$

(b) (6 pts) ...the sum of the rolls is at least 11.

$$F = \{ (5,6), (6,5), (6,6) \}$$

$$P(F) = \frac{3}{36}$$

(c) (6 pts) ...the sum of the rolls is at least 5.

$$G = \{ (1,1), (1,2), (2,1), (2,3), (2,2), (3,1) \}$$

$$P(G') = 1 - P(G) = 1 - \frac{6}{36} = \frac{30}{36}$$

(d) (6 pts) ...you roll doubles (i.e. both rolls are the same).

$$H = \{ (1,1), (2,2), \dots, (6,6) \}$$

$$P(H) = \frac{6}{36}$$

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5. You are picking four random books off of your shelf to loan to a friend. Since you are loaning them to the same friend, you don't care in which order you pull them off the shelf. Your shelf has three different series on it: the *Harry Potter* series (which has 7 books), *The Stormlight Archives* series (which has 3 books), and *The Kingkiller Chronicles* series (which has 2 books).

- (a) (12 pts) Find the probability that all four books come from the *Harry Potter* series.

$$\# \text{ of ways of picking 4 books : } {}_{12}C_4 = 495$$

$$\# \text{ of ways of picking 4 HP books : } {}_7C_4 = 35$$

$$\text{probability of picking 4 HP books : } \frac{35}{495} = .07$$

- (b) (8 pts) Find the probability that at least one book comes from *The Stormlight Archives* series.

$$\# \text{ of ways of picking no SA books : } {}_9C_4 = 126$$

$$\text{probability of picking no SA books : } \frac{126}{495}$$

$$\text{probability of picking at least one SA book : } 1 - \frac{126}{495} = .75$$

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- (c) (18 pts) Find the probability that either exactly one book comes from *The Kingkiller Chronicles* or exactly two books come from *The Stormlight Archives*.

Reminder: *Harry Potter* has 7 books, *The Stormlight Archives* has 3 books, and *The Kingkiller Chronicles* has 2 books.

$$\begin{aligned} \# \text{ of ways of choosing exactly one KC book: } & {}_2C_1 \cdot {}_{10}C_3 \\ & = 240 \end{aligned}$$

$$\begin{aligned} \# \text{ of ways of choosing exactly two SA books: } & {}_3C_2 \cdot {}_9C_2 \\ & = 108 \end{aligned}$$

$$\begin{aligned} \# \text{ of ways of choosing one KC and two SA} \\ \text{books: } & {}_2C_1 \cdot {}_3C_2 \cdot {}_7C_1 = 42 \end{aligned}$$

$$\begin{aligned} \# \text{ of ways of choosing one KC or two SA} \\ \text{books: } & 240 + 108 - 42 = 306 \end{aligned}$$

$$\begin{aligned} \text{probability of choosing one KC or two SA} \\ \text{books: } & \frac{306}{495} \end{aligned}$$