

Exam 1

Math 105, Summer 2019

Name: _____

They

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Show your work. If you give me an answer without any kind of demonstration of how you got that answer, you will not receive credit for that part of the problem.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 121 points on this exam. That means you should budget about 0.4 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder. There are to be no calculators or devices with internet access used in conjunction with this test. If you use any such material, you will receive a zero on this assessment.

1. Vocabulary (5 points each)

(a) What is a syllogism?

A syllogism is a collection of statements, called premises, followed by another statement, called the conclusion

(b) What does it mean for an argument to be valid?

An argument is valid if it is impossible for the premises to be true and the conclusion false

(c) What is a simple statement?

A simple statement is a statement which contains no logical connectives

(d) What does it mean for two statements to be logically equivalent?

Two statements are logically equivalent if they have identical columns in their truth tables

2. (5 points each) Determine if each of the following arguments is inductive or deductive and explain your answer using a complete sentence.

- (a) 1. Greg's first lecture was terrible.
2. Greg's second lecture was terrible.

∴ All of Greg's lectures are terrible.

The argument is inductive because it reasons from the specific cases of Greg's first two lectures to a general principle about all of Greg's lectures.

- (b) 1. Every computer is powered by logic.
2. Your brain is powered by logic.

∴ Your brain is a computer.

The argument is deductive because it reasons from a general claim about computers to a specific claim about your brain.

- (c) 1. Some watches are gold.
2. I have a watch.

∴ My watch is gold.

The argument is deductive because it reasons from a general claim about the set of watches to a specific claim about my watch.

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3. (10 points each) Determine if each of the following arguments is valid or invalid. Support your argument with a Venn diagram or complete sentence as appropriate.

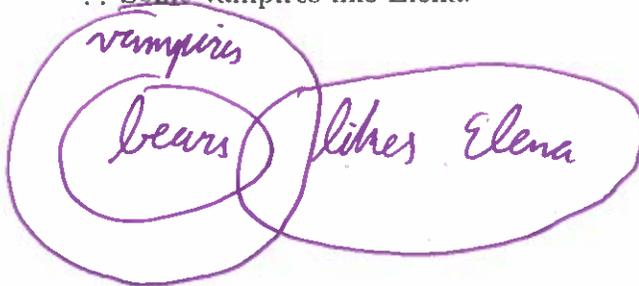
- (a) 1. Greg's first lecture was terrible.
2. Greg's second lecture was terrible.

∴ All of Greg's lectures are terrible.

This argument is inductive and therefore, invalid

- (b) 1. Some bears like Elena.
2. All bears are vampires.

∴ Some vampires like Elena.



valid

- (c) 1. No sea turtle is a fish.
2. All sea turtles are reptiles.

∴ No reptile is a fish.



invalid

4. Use the following symbolic representations for the remainder of the problem.

w : You are a great writer.

h : You are a great historian.

m : You are a great mathematician.

(a) (5 points each) Translate the following statements from symbolic logic into natural language.

i. $(m \wedge h) \rightarrow w$

If you are a great mathematician and historian,
then you are a great writer

ii. $w \vee (h \wedge \sim m)$

Either you are a great writer, or you are a great
historian but not a great mathematician

(b) (5 points each) Translate the following natural language sentences into the language of symbolic logic. If you need additional symbols, be sure to define them.

i. Being a great writer is necessary for being a great athlete.

a : You are a great athlete

$$a \rightarrow w$$

ii. You are not a great mathematician only if you are a great writer.

$$\sim m \rightarrow w$$

iii. You are a great historian, but not a great mathematician, if and only if you are a great writer and athlete.

$$(h \wedge \sim m) \leftrightarrow (w \wedge a)$$

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5. (8 points each) Analyze the truth values of the following statements.

(a) $d \rightarrow (r \vee \sim d)$

d	r	$\sim d$	$r \vee \sim d$	$d \rightarrow (r \vee \sim d)$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The statement is only false when d is true and r is false

(b) $\sim((p \wedge g) \vee \sim g)$

p	g	$p \wedge g$	$\sim g$	$(p \wedge g) \vee \sim g$	$\sim((p \wedge g) \vee \sim g)$
T	T	T	F	T	F
T	F	F	T	T	F
F	T	F	F	F	T
F	F	F	T	T	F

The statement is only true when p is false and g is true

6. (5 points) Express the contrapositive of the following statement: "Snow is sufficient for it being cold outside."

If it isn't cold outside, then it won't snow

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7. (10 points) Is the following argument valid or invalid?

1. $p \vee q$

2. $(\sim q) \rightarrow r$

$\therefore (\sim r) \rightarrow p$

Goal: $((p \vee q) \wedge (\sim q \rightarrow r)) \rightarrow (\sim r \rightarrow p)$

p	q	r	$p \vee q$ (1)	$\sim q$	$\sim q \rightarrow r$ (2)	$(1) \wedge (2)$	$\sim r$	$\sim r \rightarrow p$	$((1) \wedge (2)) \rightarrow \therefore$
T	T	T	T	F	T	T	F	T	T
T	T	F	T	F	T	T	T	T	T
T	F	T	T	T	T	T	F	T	T
T	F	F	T	T	F	F	T	T	T
F	T	T	T	F	T	T	F	T	T
F	T	F	T	F	T	T	T	F	F
F	F	T	T	T	T	T	F	T	T
F	F	F	F	T	F	F	T	T	T

invalid

Bonus: (5 possible points) State the Principle of Explosion. Why is it true?

The Principle of Explosion states that every statement follows from a contradiction

It is true because the argument

1. P
 2. $\sim P$ is valid.
- $\therefore Q$