Bonus Topics

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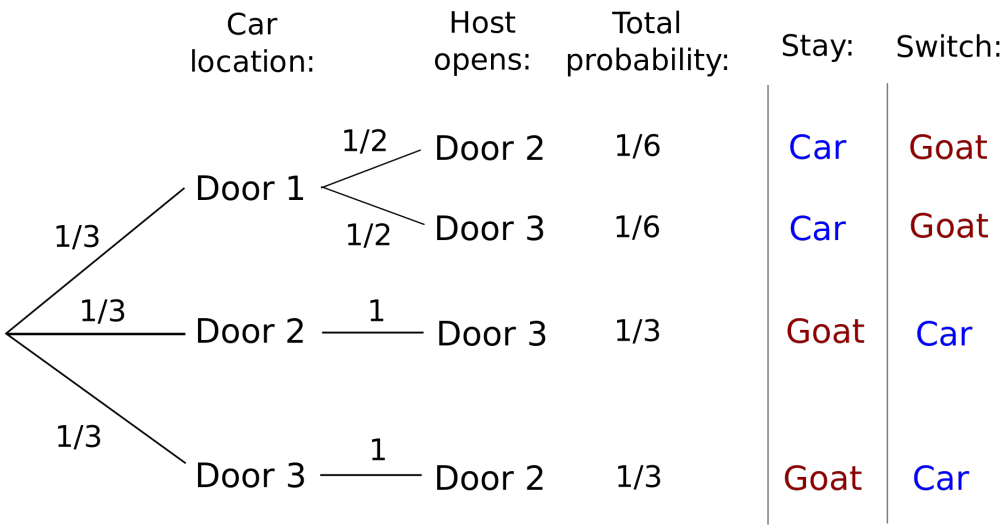
1 Probability Problems

1.1 Monty Hall Problem

- Here's the game: you are shown three doors.
  - Behind one of them is a car, and behind two of them are goats. The distribution is random.
  - You pick one door. Of the remaining doors, at least one has a goat behind it.
  - Monty Hall randomly selects a door that has a goat behind it and reveals it to you.
  - You are given the option of switching doors.
  - Should you?
- First, let's play.
- Have people pair up and hand three cards to each pair: two black and one red (or vice versa).
- Have half the room play with a switching strategy and half the room play with the staying strategy.
- Try to get at least 100 trials for each strategy in.
- Explanation 1
  - The door you pick has a 1/3 chance that it has a car behind it.
  - The other two have a 2/3 chance that the car is behind one of them.
  - Once you know that, say, door 2 doesn't have the car, the other two doors still have a 2/3 chance of the car being behind one of them, but that chance must be only apply to door 3.
- Explanation 2
  - Assuming you pick door 1...
  - By Rick Block - Own work, Public Domain, `https://commons.wikimedia.org/w/index.php?curid=3822595`



- Explanation 3
  - Another good way of thinking about this problem is to increase the number of doors
  - Say you have a million doors, one car, and the rest are goats.
  - You pick your door and Monty Hall reveals goats in all but one of the remaining doors.
  - It's pretty clear now that the one he didn't reveal almost has to have the car behind it.
- Of note is that Erdos didn't believe that you should switch until he saw a computer simulation of it.

1.2 The Baltimore Stockbroker

- Say you get a letter in the mail from someone who claims to be a stockbroker
- They say you should invest with them and to prove their knowledge of the market, they predict that a certain stock will rise.
- You check and it does.
- You're skeptical. You know that if they were randomly guessing, they'd have a 50% chance of getting it right. So you elect not to invest with them.
- Next week, the same thing happens.
- And the week after that.
- And so on, until, for 10 consecutive weeks, you get correct predictions.
- Should you invest with them? Have they proved their knowledge of the market?
- Solution
  - Suppose on week 1, they sent out 1024 letters. Half said that the stock would rise, half said it would fall.
  - Week 2, they send out letters to the 512 people who got the correction right last week. Half of those letters say the stock will rise, half say it will fall.
  - Etc.
- What's the probability that someone gets 10 consecutive correct predictions? 1.
- What's the probability that you get 10 consecutive correct predictions? 1/1024.

1.3 Where the Bullet Holes Aren't

- The problem:
  - It's WWII
  - You don't want planes getting shot down by the enemy, so you armor them
  - Too much armor slows down your planes and uses more fuel, so you want to be judicious about where you put the armor.
- The data:
  - When planes come back from Europe, they have bullet holes
  - Some parts of the plane have more bullet holes than others

Section of plane	Bullet holes per square foot
Engine	1.11
Fuselage	1.73
Fuel System	1.55
Rest of Plane	1.8

- Where to put the armor?
- Put it where the bullet holes aren't!
- This is an example of survivorship bias
- We're only examining the planes that came back from Europe, i.e. the ones that survived.
- The reason there are so few bullet holes per square foot in the engines is that the planes that get shot in the engine don't come back and we don't get that data.
- So we put the armor where the bullet holes aren't!
- There is a similar sort of thing happening with, say, food allergies.
- You can ask why people have more food allergies now than they used to and it's because people with food allergies are now surviving longer.

2 Predicate Logic

- Okay, I don't have time for this right now.