Lecture Examples

Ex 1 Consider the argument

1. $p \to q$
2. $\sim q$

$\therefore \sim p$

Use a truth table to verify its validity.

Goal: $((p \to q) \wedge (\sim q)) \to \sim p$

p	q	$p \to q$	$\sim q$	$(p \to q) \wedge (\sim q)$	$\sim p$	$((1) \wedge (2)) \to \therefore$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

tautology, so the argument is valid

Ex 2 Consider the argument

1. $p \vee r$
2. $\sim q$

$\therefore q \vee r$

Use a truth table to determine if it is valid or not.

Goal: $((p \vee r) \wedge \sim q) \to q \vee r$

p	q	r	$p \vee r$	$\sim q$	$(1) \wedge (2)$	$q \vee r$	$((1) \wedge (2)) \to \therefore$
T	T	T	T	F	F	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	F	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	F	F	T

not a tautology, so the argument

Ex 3 Translate the following natural language argument into a symbolic argument. Then determine whether the argument is valid or not. " 'A dog's not mad. You grant that?' 'I suppose so,' said Alice. 'Well then,' the cat went on, 'you see a dog growls when it's angry, and wags its tail when it's pleased. Now *I* growl when I'm pleased, and wag my tail when I'm angry. Therefore, I'm mad!"'

d: It is a dog

m: It is mad

b: It growls when angry and wags its tail when pleased

Argument:

1. $d \to \sim m$
2. $d \to b$
3. $\sim b$

$\therefore m$

Goal: $((① \wedge ②) \wedge ③) \to \therefore$

d	b	∴ m	~m	① d→~m	② d→b	③ ~b	①∧②	(①∧②)∧③	((①∧②)∧③)→∴
T	T	T	F	F	T	F	F	F	T
T	T	F	T	T	T	F	T	F	T
T	F	T	F	F	F	T	F	F	T
T	F	F	T	T	F	T	F	F	T
F	T	T	F	T	T	F	T	F	T
F	T	F	T	T	T	F	T	F	T
F	F	T	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T	F

Not a tautology, so the argument is invalid

On-Your-Own Examples

Ex 1 Use the given symbols to rewrite the argument in symbolic form.
p: The senator supports new taxes.
q: The senator is reelected.

The senator is not reelected if she supports new taxes.
The senator does not support new taxes.

Therefore, the senator is reelected.

1. $p \to \sim q$
2. $\sim p$

$\therefore q$

Ex 2 Is the following statement a tautology? (Hint: Use a truth table.)

$$\sim (((p \to q) \wedge p) \wedge \sim q)$$

p	q	$p \to q$	$(p \to q) \wedge p$	$\sim q$	$((p \to q) \wedge p) \wedge \sim q$	$\sim (((p \to q) \wedge p) \wedge \sim q)$
T	T	T	T	F	F	T
T	F	F	F	T	F	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

The statement is a tautology

Ex 3 Define the necessary symbols and rewrite the statements in symbolic form. Then use a truth table to determine whether each argument is valid.

(a) If you do not recycle newspapers, you are not an environmentalist.
If you recycle newspapers, you save trees.

Therefore, you are an environmentalist only if you save trees.

1. $\sim r \to \sim e$
2. $r \to s$
$\therefore e \to s$

r: You recycle newspapers
e: You are an environmentalist
s: You save trees

Goal: $((\sim r \to \sim e) \wedge (r \to s)) \to (e \to s)$

r	e	s	$\sim r$	$\sim e$	① $\sim r \to \sim e$	② $r \to s$	① ∧ ②	$e \to s$	(① ∧ ②) → ∴
T	T	T	F	F	T	T	T	T	T
T	T	F	F	F	T	F	F	F	T
T	F	T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	F	T	F	F	T
F	F	T	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T

Invalid

(b) All forest rangers are environmentalists.
All forest rangers are storytellers.

Therefore, all environmentalists are storytellers.

1. $r \to e$
2. $r \to s$
$\therefore e \to s$

r: You are a forest ranger
e: You are an environmentalist
s: You are a storyteller

r	e	s	① $r \to e$	② $r \to s$	$e \to s$	① ∧ ②	(① ∧ ②) → ∴
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Invalid

(c) It snows only if it's freezing.
It isn't freezing.

Therefore, it doesn't snow.

1. $s \to f$
2. $\sim f$
$\therefore \sim s$

s: It snows
f: It is freezing

s	f	$\sim s$ (∴)	$\sim f$ (②)	$s \to f$ (①)	① ∧ ②	(① ∧ ②) → ∴
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Valid

(d) All Jedi are one with the force.
Yoda is one with the force.

Therefore, Yoda is a Jedi.

1. $j \to f$
2. f
$\therefore j$

j: It is a Jedi
f: It is one with the force

j	f	$j \to f$ (①)	① ∧ ②	(① ∧ ②) → ∴ (∴ ②)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	(F)
F	F	T	F	T

invalid