Section 1.4 Lecture Guide

Lecture Examples

Ex 1 Make a truth table with the statements $p \to q$ (a conditional), $q \to p$ (the inverse of the conditional), $(\sim p) \to (\sim q)$ (the converse of the conditional), and $(\sim q) \to (\sim p)$ (the contrapositive of the conditional) . Which of the statements are equivalent?

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

$p \to q \equiv \sim q \to \sim p$

$q \to p \equiv \sim p \to \sim q$

Ex 2 Consider the conditional statement "Being an athlete is necessary for being a dancer." Express the contrapositive...

If one is a dancer, then one is an athlete

(a) ...as a sufficient condition.

C.P.: If one is not an athlete, then one is not a dancer

Not being a dancer is necessary for not being an athlete

(b) ...as a necessary condition.

Not being an athlete is sufficient for not being a dancer

On-Your-Own Examples Summary of equivalences

$\sim(\sim p)$	$\equiv$	p	negation of a negation
$\sim(p \wedge q)$	$\equiv$	$(\sim p) \vee (\sim q)$	negation of a conjunction
$\sim(p \vee q)$	$\equiv$	$(\sim p) \wedge (\sim q)$	negation of a disjunction
$\sim(p \to q)$	$\equiv$	$p \wedge (\sim q)$	negation of a conditional
$p \to q$	$\equiv$	$(\sim q) \to (\sim p)$	a conditional and its contrapositive
$q \to p$	$\equiv$	$(\sim p) \to (\sim q)$	the converse and inverse of a conditional
$p \leftrightarrow q$	$\equiv$	$(p \to q) \wedge (q \to p)$	biconditional

Ex 1 Given the following statements, write the natural language sentence represented by the symbols.
p: I am sleeping.
q: I am snoring.

(a) $p \to q$

If I am sleeping, then I am snoring

(b) $q \to p$

If I am snoring, then I am sleeping

(c) $(\sim p) \to (\sim q)$

If I am not sleeping, then I am not snoring

(d) $(\sim q) \to (\sim p)$

If I am not snoring, then I am not sleeping

(e) $p \leftrightarrow q$

I am sleeping if and only if I am snoring

Ex 2 Write an equivalent variation of the given conditional.

(a) If it is not snowing, then it is warm outside.

If it is not warm outside, then it is snowing

(b) You are not a vegetarian if you eat meat.

If you are a vegetarian, then you don't eat meat

Ex 3 State the converse of the sentence: If she goes to the store, she will buy ice cream.

If she doesn't go to the store, she will not buy ice cream

Ex 4 Translate the two sentences into symbolic form and ~~use a truth table to~~ determine whether the statements are equivalent.

(a) Being an automobile that is American-made is sufficient for an automobile having hardware that is not metric.

(b) Being an automobile that is not American-made is necessary for an automobile having hardware that is metric.

a: An automobile is American-made

h: An automobile has metric hardware

(a) $a \rightarrow \sim h$

(b) $h \rightarrow \sim a$

The statements are contrapositives, so they are equivalent