

Lecture Examples

Ex 1 A local 4-H club surveyed its members, and the following information was obtained: 13 members had rabbits, 10 members had goats, 4 had both rabbits and goats, and 18 had neither rabbits nor goats.

(a) What percent of the club members had rabbits or goats?

$$R = \{x \mid x \text{ has rabbits}\} \quad n(R) = 13, \quad n(G) = 10, \quad n(R \cap G) = 4$$

$$G = \{x \mid x \text{ has goats}\} \quad n((R \cup G)') = 18$$

$$n(R \cup G) = n(R) + n(G) - n(R \cap G) = 13 + 10 - 4 = 19$$

$$n(U) = n(R \cup G) + n((R \cup G)') = 19 + 18 = 37 \Rightarrow \frac{19}{37} \approx 51\%$$

(b) What percent of the club members had only rabbits?

13 have rabbits, 4 have both, so 9 have only rabbits

$$\Rightarrow \frac{9}{37} \approx 24\%$$

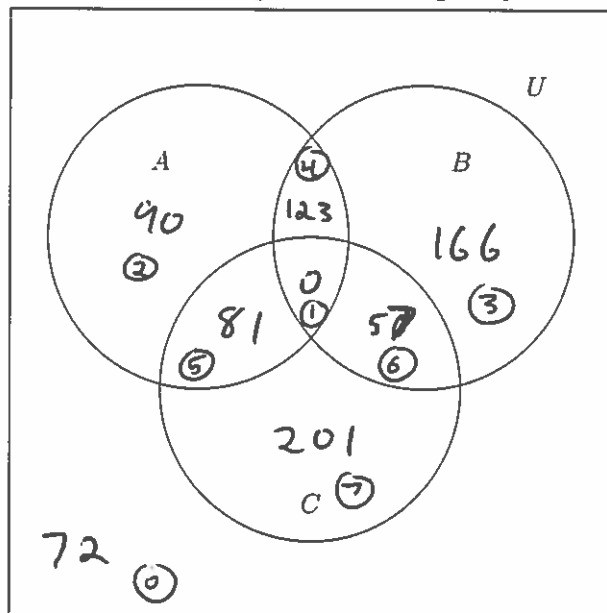
(c) What percent of the club members had only goats?

10 have goats, 4 have both, so 6 have only goats

$$\Rightarrow \frac{6}{37} \approx 16\%$$

Ex 2 After leaving the polls, many people are asked how they voted. (This is called an *exit poll*). Concerning Propositions a, b, and c, the following information was obtained: 294 people voted yes on a, 346 voted yes only on a, 166 voted yes on b, 166 voted yes only on b, 517 voted yes on a or b, 339 voted yes on c, no one voted yes on all three, and 72 voted no on all three.

(a) Fill out the Venn diagram illustrating the poll results.



$$A = \{x \mid x \text{ voted yes on a}\}$$

$$B = \{x \mid x \text{ voted yes on b}\}$$

$$C = \{x \mid x \text{ voted yes on c}\}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$517 = 294 + 346 - n(A \cap B)$$

$$= 640 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 123$$

(b) What percent of the voters in the exit poll voted no on A?

$$n(U) = 790 \Rightarrow \frac{496}{790} \approx 63\%$$

$$n(A') = 496$$

(c) What percent of the voters voted yes on more than one proposition?

$$123 + 81 + 57 = 261 \Rightarrow \frac{261}{790} \approx 33\%$$

Ex 3 Let $U = \{0, 1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$.

(a) For the given sets, compare $(A \cup B)'$ and $A' \cup B'$.

$$\begin{aligned} A \cup B &= \{1, 2, 3, 4, 5\} & A' &= \{0, 4, 5\} & B' &= \{0, 1, 3\} \\ (A \cup B)' &= \{0\} & A' \cup B' &= \{0, 1, 3, 4, 5\} \end{aligned}$$

(b) For the given sets, compare $(A \cap B)'$ and $A' \cup B'$.

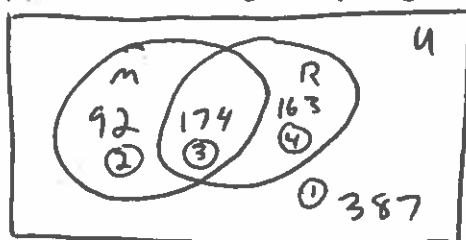
$$\begin{aligned} A \cap B &= \{2\} \\ (A \cap B)' &= \{0, 1, 3, 4, 5\} \\ (A \cap B)' &= A' \cup B' \end{aligned}$$

On-Your-Own Examples

Ex 4 A survey asked 816 college freshman whether they had been to a movie or eaten in a restaurant during the past week. The following information was obtained: 387 freshman had been to neither a movie nor a restaurant, 266 had been to a movie, and 92 of those who had been to a movie had not been to a restaurant.

$$M = \{x \mid x \text{ has been to a movie}\} \quad R = \{x \mid x \text{ has eaten at a restaurant}\}$$

(a) Draw a Venn diagram depicting the survey results.



$$\begin{aligned} n(U) &= 816 \\ n(M' \cap R') &= 387 \\ n(M) &= 266 \\ n(M \cap R) &= 92 \end{aligned}$$

(b) How many of the surveyed freshman had been to both a movie and a restaurant?

$$\frac{92}{816} \approx 11\%$$

(c) How many of the surveyed freshman had been to a movie or a restaurant?

$$\frac{92 + 174 + 163}{816} \approx 53\%$$

(d) How many of the surveyed freshman had been to a restaurant?

$$174 + 163 = 337$$

(e) How many of the surveyed freshman had been to a restaurant but not a movie?

$$163$$

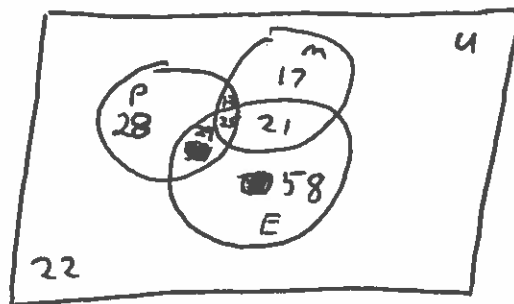
Ex 5 In a recent survey of monetary donations made by college graduates, the following information was obtained: 95 graduates had donated to a political campaign, 76 had donated to assist medical research, 133 had donated to help preserve the environment, 25 had donated to all three, 22 had donated to none of the three, 38 had donated to a political campaign and to medical research, 46 had donated to medical research and to preserve the environment, and 54 had donated to a political campaign and to preserve the environment.

$P = \{x \mid x \text{ donated to a political campaign}\}$, $M = \{x \mid x \text{ donated to medical research}\}$, $E = \{x \mid x \text{ donated to help preserve the environment}\}$

(a) Draw a Venn diagram depicting the monetary donations made by the college graduates.

$$\begin{aligned} n(P) &= 95 \\ n(M) &= 76 \\ n(E) &= 133 \\ n(P \cap M \cap E) &= 25 \\ n(P' \cap M' \cap E') &= 22 \\ n(P \cap M) &= 38 \end{aligned}$$

$$\begin{aligned} n(M \cap E) &= 46 \\ n(P \cap E) &= 54 \end{aligned}$$



(b) What percent of the college graduates donated to none of the three listed causes?

$$n(U) = 22 + 28 + 13 + 25 + 17 + 21 + 58 = 213$$

$$\frac{22}{213} \approx 10.3\%$$

(c) What percent of the college graduates donated to exactly one of the three listed causes?

$$28 + 58 + 17 = 103$$

$$\frac{103}{213} \approx 48\%$$

Ex 6 Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{0, 1, 3, 8\}$, and $B = \{0, 4, 6, 8, 9\}$.

(a) Find $A \cap B$. = $\{0, 8\}$

(b) Find A' . = $\{2, 4, 5, 6, 7, 9\}$

(c) Find B' . = $\{1, 2, 3, 5, 7\}$

(d) Use De Morgan's Laws to find $(A' \cap B)'$. = $A'' \cup B' = A \cup B'$

$$= \{0, 1, 3, 8, 2, 5, 7\}$$

(e) Use De Morgan's Laws to find $(A \cup B)'$. = $A' \cap B' = A' \cap B$

$$= \{4, 6, 9\}$$

