Lecture Examples

Ex 1 Let U be the set of all real numbers. Consider the sets $R = \{1, \pi, 10\}$, $S = \{3, 9\}$, and $T = \{x \mid x \text{ is a whole number between } -3 \text{ and } 500\}$.

(a) What is $R \cup S$?

$R \cup S = \{1, \pi, 10, 3, 9\}$

(b) What is $R \cap T$?

$R \cap T = \{1, 10\}$

(c) What is $R \cap S$?

$R \cap S = \emptyset$

Ex 2 With universe U being the set of all whole numbers, consider the sets $A = \{1, -3, 4, -2, 0\}$ and $B = \{3, 2, 0, 1\}$, find

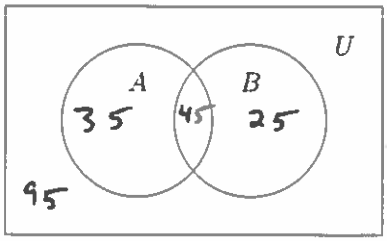
(a) $A \cap B = \{0, 1\}$

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 4 - 2 = 7$

Ex 3 Given $n(U) = 200$, $n(A) = 80$, and $n(B) = 70$, do the following:

(a) If $n(A \cap B) = 45$, find $n(A \cup B)$ and draw a Venn diagram illustrating the composition of the universal set U.

$n(A \cup B) = 105$



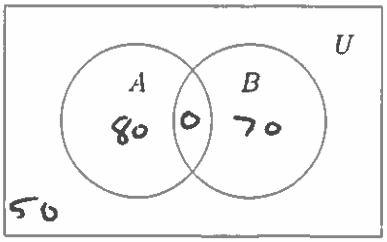
(b) If $n(A \cup B) = 150$, find $n(A \cap B)$ and draw a Venn diagram illustrating the composition of the universal set U.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$150 = 80 + 70 - n(A \cap B)$

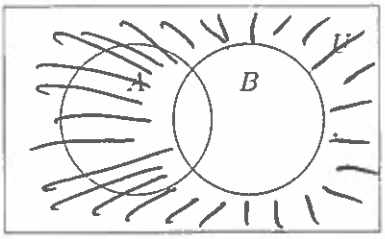
$= 150 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 0$

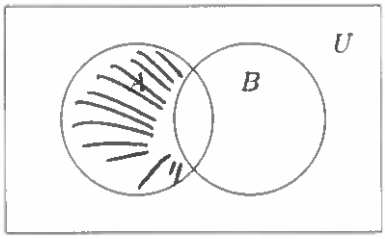


Ex 4 In each Venn diagram, shade the given set.

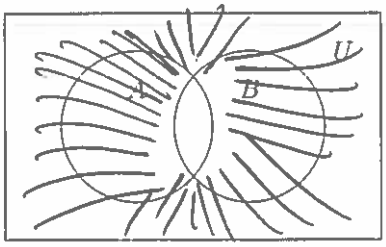
(a) B'



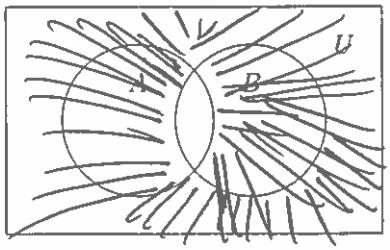
(b) $A \cap B'$



(c) $(A \cap B)'$



(d) $A' \cup B'$



On-Your-Own Examples

Ex 1 If $U = \{0,1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$, $B = \{7,8,9\}$, and $C = \{1,3,5\}$, find the following:

(a) $A \cap B = \{8\}$

(b) $B \cap C = \emptyset$

(c) $(A \cap B) \cap C = \emptyset$

(d) $A \cup B = \{2,4,6,7,8,9\}$

(e) $B \cup C = \{1,3,5,7,8,9\}$

(f) $A' = \{0,1,3,5,7,9\}$

(g) $B' = \{0,1,2,3,4,5,6\}$

(h) $C' = \{0,2,4,6,7,8,9\}$

(i) $B \cap C' = \{7,8,9\}$

(j) $A' \cap B' = \{0,1,3,5\}$

(k) $(A \cup B)' = \{0,1,3,5\}$

Ex 2 Suppose $n(U) = 120$, $n(A) = 45$, and $n(B) = 80$.

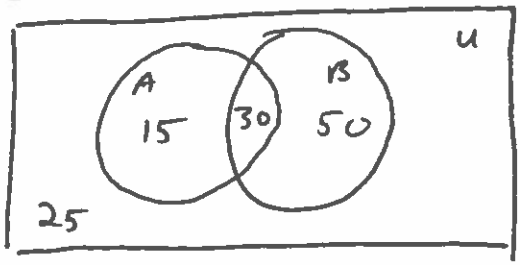
(a) If $n(A \cup B) = 95$, find $n(A \cap B)$ and draw a Venn diagram illustrating the sets A and B in U.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

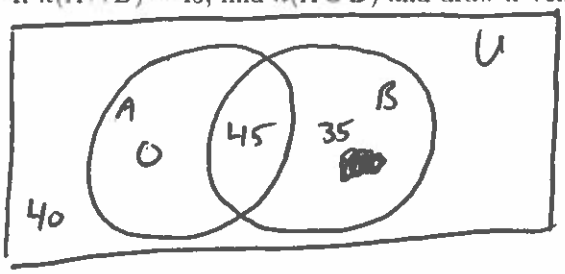
$95 = 45 + 80 - n(A \cap B)$

$= 125 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 30$



(b) If $n(A \cap B) = 45$, find $n(A \cup B)$ and draw a Venn diagram illustrating the sets A and B in U.



Ex 3 A transportation survey of 300 college students (the universal set U) yielded the following information: 211 students own a bicycle, 145 students own an automobile, and 34 students own neither.

(a) How many students in the sample own either a bicycle or an automobile?

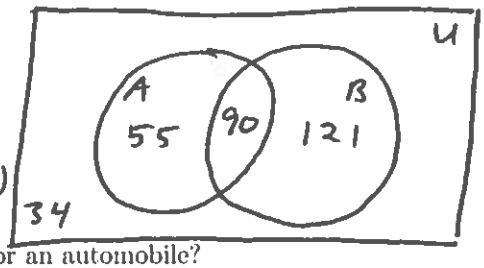
$B = \{x \mid x \text{ owns a bicycle}\}$

$A = \{x \mid x \text{ owns an automobile}\}$

$n(A \cup B) = 300 - 34 = 266$

$266 = n(A) + n(B) - n(A \cap B) = 145 + 211 - n(A \cap B)$

$= 356 - n(A \cap B) \Rightarrow n(A \cap B) = 90$



(b) What percentage of students own either a bicycle or an automobile?

$n(A \cup B) = 55 + 90 + 121 = 266 \Rightarrow \frac{266}{300} \approx .89 \Rightarrow 89\%$

(c) How many students own a bicycle but not an automobile?

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