Recommended 2:4 Problems

2.4 (1-4, 17, 18, 20, 27, 28)

2a) $_8P_4 = \dfrac{8!}{(8-4)!} = 1680$

b) $_8C_4 = \dfrac{8!}{(8-4)!\,4!} = 70$

4a) $_9P_0 = \dfrac{9!}{(9-0)!} = 1$

b) $_9C_0 = \dfrac{9!}{(9-0)!\,0!} = 1$

8a) Drawing is done without replacement
Since order is important, there are
$_{23}P_3 = 10,626$ possibleorderings of the speakers

b) Now, order is not important, so there are
$_{23}C_3 = 1,771$ ways of choosing the speakers

20a) If the members ~~are~~ of the committee all do the same thing, then the order of their selection is unimportant and there are $_{16}C_4 = 1820$ ~~ill~~ different committees

b) If there is a chairperson and the rest are general members, there are two categories:

$$\underset{\text{chair}}{\underline{_{16}C_1}} \quad \underset{\text{others}}{\underline{_{15}C_3}} \quad \rightarrow \quad \begin{aligned} _{16}C_1 &= 16 \\ _{15}C_3 &= 455 \end{aligned}$$

Hence, there are $16 \cdot 455 = 7280$ possible committees

c) If there are 4 distinct roles, then order is important and there are $_{16}P_4 = 43,680$ possible committees.

You could also think about this as having four categories: $\underset{\text{chair}}{\underline{16}} \quad \underset{\text{secretary}}{\underline{15}} \quad \underset{\text{refreshments}}{\underline{14}} \quad \underset{\text{cleaning}}{\underline{13}}$

leading to $16 \cdot 15 \cdot 14 \cdot 13 = 43,680$ possible committees

27) There are ${}_{52}C_5 = 2{,}598{,}960$ possible 5 card hands since order does not matter and drawing is done without replacement

28a) There are ${}_{13}C_5 = 1287$ hands with all hearts

b) As in part (a), there are 1287 hands with all diamonds, 1287 hands of all clubs, and 1287 hands of all spades.

Hence, there are $4 \cdot 1287 = 5148$ hands of all of the same suit