Lecture Examples

Ex 1 If there are 21 students in a class, what's the probability that two or more students share the same birthday? Assume that birthdays are assigned randomly to each person and that there are 365 days in a year.

# of ways that no one shares a birthday: ${}_{365}P_{21}$

# of ways that birthdays can be distributed: 365^{21}

probability that no one shares a birthday:

$$\frac{{}_{365}P_{21}}{365^{21}} \approx .55$$

probability that two or more students share a birthday:

$1 - .55 = .45$

Ex 2 What's the probability that *you* share a birthday with someone in the room?

# of ways that no one else's birthday is 2/4: 364^{20}

# of ways that birthdays can be distributed among people who aren't me: 365^{20}

probability that no one else's birthday is 2/4: $\frac{364^{20}}{365^{20}} \approx .95$

probability that at least one other person has birthday on 2/4: $1 - .95 = .05$

Ex 3 In a 6/44 lottery, a player selects 6 (different) numbers between 1 and 44. If the player's selections match the six winning numbers, the player wins first prize. If five out of the six match, the player wins second prize. What is the probability of winning first prize? What's the probability of winning second prize?

# of possible winning combinations : ${}_{44}C_6$

# of ways of getting 5 of 6 right : ${}_6C_5 \cdot {}_{38}C_1 = 228$
(winning, not)

probability of winning first prize : $\frac{1}{{}_{44}C_6} \approx .0000001$

probability of winning second prize : $\frac{228}{{}_{44}C_6} \approx .00003$

Ex 4 A different type of lottery is the Powerball. Powerball involves selecting five (different) numbers from 1 to 59 and then a "powerball number," which is a number from 1 to 39. A player wins first prize if all six numbers match the drawing. What is the probability of picking the winning numbers?

# of possible winning combinations : ${}_{59}C_5 \cdot 39$

probability of picking all the winning numbers : $\frac{1}{{}_{59}C_5 \cdot 39} \approx .00000005$

Ex 5 A 5-card hand is dealt from a standard 52-card deck.

(a) Find the probability of being dealt ~~3~~ at least 3 aces.

# of ways of being dealt 3 aces: ${}_4C_3 \cdot {}_{49}C_2 = 4704$

# of ways of being dealt 5 cards: ${}_{52}C_5 = 2598960$

probability of being dealt at least 3 aces: $\frac{4704}{2598960} \approx .002$

(b) What is the probability of being dealt the 3 of clubs and the ace of diamonds?

# of ways of being dealt the 3 of clubs and ace of diamonds: $1 \cdot 1 \cdot {}_{50}C_3 = 19{,}600$

probability of being dealt the 3 of clubs and ace of diamonds: $\frac{19600}{2598960} \approx .007$

(c) Find the probability of being dealt ~~a pair of~~ exactly two queens or ~~a pair of~~ exactly two kings.

# of hands with exactly two queens: ${}_4C_2 \cdot {}_{48}C_3 = 103776$

# of hands with exactly two kings: ${}_4C_2 \cdot {}_{48}C_3 = 103776$

# of hands with exactly two kings and exactly two queens: ${}_4C_2 \cdot {}_4C_2 \cdot {}_{44}C_1 = 1584$

# of hands with two queens or two kings: $103776 + 103776 - 1584 = 205968$

probability of being dealt exactly two queens and two kings: $\frac{205968}{2598960} \approx .079$

On-Your-Own Examples

Ex 1 Find the probability of winning first prize in a 6/41 lottery.

# of winning combinations : ${}_{41}C_6 = 4496388$

probability of winning 1st prize: $\frac{1}{4496388} \approx .0000002$

Ex 2 Find the probability of winning second prize (5 matching numbers) in a 6/41 lottery.

# of ways of getting 5 numbers correct: ${}_6C_5 \cdot {}_{41}C_1 = 246$

probability of winning 2nd prize: $\frac{246}{4496388} \approx .00005$

Ex 3 Three six-sided dice are rolled. Find the probability of getting triples.

$S = \{(1,1,1), (1,1,2), (1,2,1), (2,1,1), \ldots\}$

$n(S) = 6^3 = 216$

$T = \{(1,1,1), (2,2,2), (3,3,3), \ldots, (6,6,6)\}$

$P(T) = \frac{n(T)}{n(S)} = \frac{6}{216} \approx .03$

Ex 4 A 5-card hand is dealt from a standard 52-card deck.

(a) Find the probability of being dealt ~~at least~~ exactly two aces.

# of hands with at least two aces: ${}_4C_2 \cdot {}_{48}C_3 = 103776$

probability of being dealt at least two aces: $\frac{103776}{2598460} \approx .04$

(b) Find the probability that the hand consists of at least two aces or at least three jacks.

# of hands with at least 3 jacks: ${}_4C_3 \cdot {}_{49}C_2 = 4704$

# of hands with at least 3 jacks and at least two aces: ${}_4C_3 \cdot {}_4C_2 = 24$

↳ only way of this happening is with 3 jacks and two aces

# of hands with at least 2 aces or at least 3 jacks: $117,600 + 4704 - 24 = 122280$

probability of being dealt at least two aces or at least 3 jacks: $\frac{122280}{2598460} \approx .05$

Ex 5 Suppose that four faulty transistors are accidentally packaged with sixteen reliable ones. If three transistors are chosen at random from the package, what is the likelihood that:

(a) all three are reliable?

# of ways of choosing 3 transistors: ${}_{20}C_3 = 1140$

# of ways all three are reliable: ${}_{16}C_3 = 560$

probability all 3 are reliable: $\frac{560}{1140} \approx .49$

(b) none are reliable?

# of ways none are reliable: ${}_4C_3 = 4$

probability none are reliable: $\frac{4}{1140} \approx .004$

(c) exactly one is reliable?

# of ways exactly one is reliable: ${}_{16}C_1 \cdot {}_4C_2 = 96$
(reliable, not)

probability exactly one is reliable: $\frac{96}{1140} \approx .08$

(d) at least one is reliable?

↳ this is the complement to the event in (b)

$1 - .004 = .996$