Lecture Examples

Ex 1 Under what conditions is the sentence "It is not the case that I both play basketball and golf" true?

b: I play basketball

g: I play golf

$\sim(b \wedge g)$: It is not the case that I both play basketball and golf.

b	g	$b \wedge g$	$\sim(b \wedge g)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The sentence is true if I don't play basketball and I play golf, if I play basketball and I don't play golf, or if I play neither sport.

Ex 2 Under what conditions is the sentence "Either I do not play basketball or I do not play golf" true?

$\sim b \vee \sim g$: Either I do not play basketball or I do not play golf

b	g	$\sim b$	$\sim g$	$\sim b \vee \sim g$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The sentence is true under the same conditions as the sentence in Ex 1.

On-Your-Own Examples

Ex 1 Analyze the truth values of the symbolic statement $q \to \sim p$.

p	q	$\sim p$	$q \to \sim p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

$q \to \sim p$ is false only when p and q are both false

Ex 2 Construct a truth table for the symbolic expression $p \vee \sim p$.

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Ex 3 Analyze the truth values of the symbolic statement $p \wedge \sim (q \vee r)$

p	q	r	$q \vee r$	$\sim (q \vee r)$	$p \wedge \sim (q \vee r)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

$p \wedge \sim (q \vee r)$ is only true in the case when p is true, q is false, and r is false

Ex 4 Construct a truth table for the symbolic expression $(\sim r \vee p) \rightarrow (q \wedge p)$.

p	q	r	$\sim r$	$\sim r \vee p$	$q \wedge p$	$(\sim r \vee p) \rightarrow (q \wedge p)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
T	F	T	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	T	T	F	F
F	F	T	F	F	F	T
F	F	F	T	T	F	F

Ex 5 Compare the truth values of the symbolic expressions $p \wedge \sim q$ and $p \rightarrow q$.

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

The truth values are opposite so $\sim(p \rightarrow q) \equiv (p \wedge \sim q)$

Ex 6 Construct a truth table for the following compound statement: "I walk up the stairs if I want to exercise or if the elevator isn't working."

s : I walk up the stairs
e : I want to exercise
w : The elevator is working

$(e \vee \sim w) \to s$: I walk up the stairs if I want to exercise or if the elevator isn't working

s	e	w	$\sim w$	$e \vee \sim w$	$(e \vee \sim w) \to s$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	T
F	F	F	T	T	F

Ex 7 Construct a truth table to determine whether the following statements are equivalent.

$n \vee \sim p$: The Reds won 90 games or the Reds didn't make the playoffs.

$\sim n \to \sim p$: If the Reds did not win 90 games, then the Reds did not make the playoffs.

n : The Reds won 90 games
p : The Reds made the playoffs

n	p	$\sim n$	$\sim p$	$n \vee \sim p$	$\sim n \to \sim p$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

The statements are equivalent

Ex 8 Apply De Morgan's Laws to the following statements (leave your answer in natural language):

(a) It is not true that Marco is on both the soccer team and the tennis team.

Marco is not on the soccer team or Marco is not on the tennis team

(b) It is not true that Martha plays piano or violin.

Martha does not play piano and Martha does not play violin